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On the congruent transformation of electrical networks for equivalent forms

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ON THE CONGRUENT TRANSFORMATION OF ELECTRICAL NETWORKS FOR EQUIVALENT FORMS

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CONGRUENT TRANSFORMATION OF ELECTRICAL NETWORKS
FOR EQUIVALENT FORMS

by

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ABSTRACT

In this thesis the generation of equivalent networks by means of a congruent transformation using a variable transformation matrix applied to the admittance matrix of an n+1 node, common datum, network is studied. The ranges of the values of the elements of the transformation matrix which yield passive RLC components for the equivalent network without mutual inductances are considered. When the transformation matrix is diagonal the solution involves a set of linear algebraic inequalities. In the general case, when all the elements of the transformation matrix are present, the solution involves non-linear algebraic inequalities. The region of passive component realizability consists of a hyper-volume in the space of the elements of the transformation matrix, and the use of exponential searching functions to generate equivalent networks with design constraints is investigated.

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Equivalent Networks - Historical Background

1.1 Introduction

It is well known that there is no unique solution to a synthesis problem for realizing a given driving point or transfer function. A group of networks can always be constructed which have the same input immittance, or transfer function, or both. These groups of networks are called equivalent networks for the particular network function of interest. In general, equivalent networks can be generated by two distinct approaches.

(a) Using different synthesis procedures for realizing the network function, such as Brune, Darlington, Bott-Duffin, Miyata or other procedures for the case of input impedance synthesis. These procedures are well covered in the literature for input as well as transfer function synthesis, Ginllemin [3], Van Valkenburg [8].

(b) Using a proper linear transformation of the admittance or impedance matrix of a given network such that the immittance between any desired node pair, or the transfer function relating the response in any node pair to the excitation at another pair, is kept invariant.

The second method is a more general approach and is readily applicable to computer design techniques. It leads to a technique for generating a group of equivalent networks among which the best suited to the problem at hand can be chosen.

The method of generating equivalent networks by linear transformation was first introduced by Cauer [1] in 1929 in Germany and the first English paper on the subject was printed in 1931 by Howitt [2]. Guillemin [3], discussing this subject, has shown that the same result as obtained by a transformation can be achieved by elementary operations on the rows and columns of the network admittance or impedance matrix. Schoeffler [4] has shown that the transformation can also be applied to the branch matrices, and has also developed a technique of generating continuously equivalent networks [5].

In this chapter a discussion of the Cauer's, Guillemin and Schoeffler's method is presented.

1.2 Equivalent Driving Point and Transfer Impedance

The equilibrium conditions of a network containing n + 1 nodes with n ports defined by the terminals between each of the n nodes and datum, can be written in the following matrix form:

$$[I] = [Y][V] \tag{1-1}$$

where:

[V] is the column matrix of the node to datum voltages

[I] is the column matrix of source currents connected between each node and datum

[Y] is the n x n node to datum admittance matrix of network

The admittance matrix is formed in the following manner assuming no mutual coupling in the network.

(1) Each diagonal element y_{ii} is the sum of all the admittances connected to node i.

(2) Each off diagonal element y_{ij} $i \neq j$ is the negative of the admittance connected between nodes i and j.

The driving point impedance of a network excited at the input node i only is given by

$$Z_i = \frac{\Delta_{ii}}{\Delta} \tag{1-2}$$

where:

Δ = the determinent of the admittance matrix [Y].

Δ_{ii} = the minor obtained from Δ by deleting the i^{th} row and the i^{th} column.

Equation (1-2) can be obtained by expanding (1-1) and using Crammer's rule.

It has been shown by Howitt [2] that if the admittance matrix [Y] undergoes a real non-singular transformation Γ

$$[Y'] = [\Gamma][Y][\Gamma]^T \tag{1-3}$$

where $[\Gamma]$ is a square real non-singular matrix of the same order as [Y] and T denotes transpose, then [Y'] is the admittance matrix of a new network with the same number of nodes as the original network.

In general the input impedance at any node i and the transfer impedance between any two ports i and j of the new network is different from that of the original network. However Howitt [2] and Schoeffler [4] have shown that if the i^{th} column of $[\Gamma]$ consists of zeros except for the i^{th} term which is unity, then the input impedance at port i remains invariant under transformation.* If two such columns are chosen, the driving point impedance at the ports corresponding to the columns, as well as the transfer impedance between them, is invariant to the transformation. Example 1-1 illustrates the invariance of the input impedance under transformation (1-3) for a simple 3 node circuit.

EXAMPLE 1-1

Consider the resistance network of Fig. 1-1a.

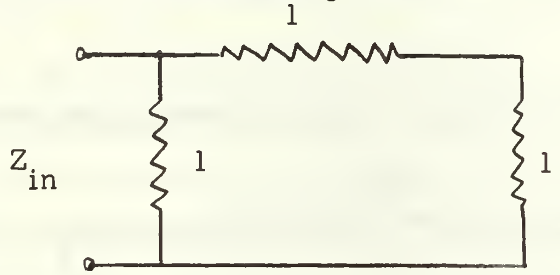

Fig. 1-1a

$$[Y] = [G] = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$Z_{in} = \frac{\Delta_{ii}}{\Delta} = \frac{2}{3}$$

Using the following transformation matrix:

$$[\Gamma] = \begin{bmatrix} 1 & a \\ 0 & b \end{bmatrix}$$ where a and b are arbitrary numbers yields

$$[G'] = \begin{bmatrix} 2(1-a-2a^2) & -b+2ab \\ -b+2ab & 2b^2 \end{bmatrix}$$

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*An original proof based on matrix partioning is given in appendix I.

resulting in Fig. 1-1b where

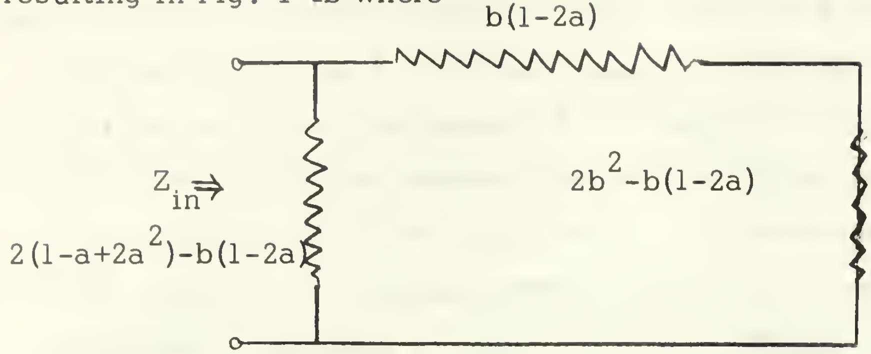

Fig. 1-1b

$$Z_{in} = \frac{\Delta_{ii}}{\Delta} = \frac{2b^2}{2b^2(1-a+a^2) - (-b+2ab)^2} = \frac{2}{3}$$

(a) Substituting $a = 3$, $b = -\frac{11}{4}$ will lead to the circuit of

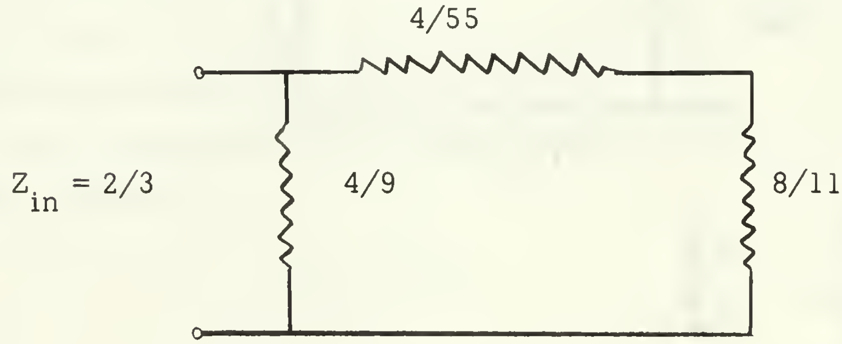

Fig. 1-1c

(b) Substituting $a = 3$, $b = -1$ will lead to the circuit of Fig. 1-1d.

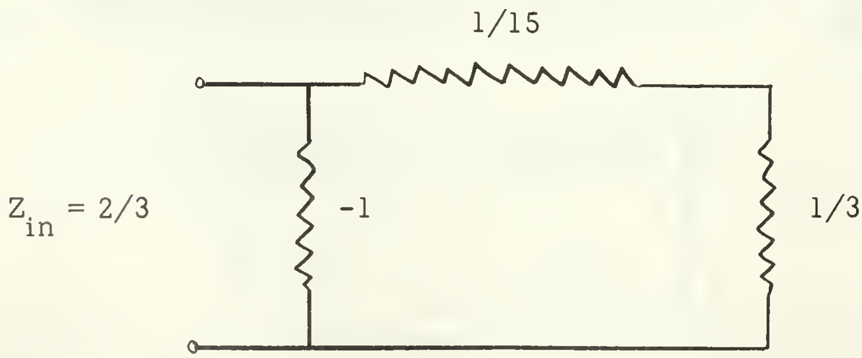

Fig. 1.1d

As can be seen the 3 networks are equivalent, in that their input impedance is the same.

As can be seen from example (1-1), the choice of the elements of $[\Gamma]$, other than the first column are arbitrary. If any of the elements in the transformed network are zero, the number of elements in the new network is different from the original network.

If the network under consideration contains inductance and capacitances as well as resistances then we can write

$$[Y] = [G] + \frac{1}{S}[L^{-1}] + S[C] \qquad (1\text{-}4)$$

where

$[G]$ = the network conductance matrix

$[C]$ = the network capacitance matrix

$[L^{-1}]$ = the network inverse inductance matrix

By the distribution law of matrix multiplication [7] the transformation $[\Gamma]$ can be applied to each of the above matrices to obtain the admittance matrix of the new network.

$$[Y'] = [G'] + \frac{1}{S}[L'^{-1}] + S[C'] \qquad (1\text{-}5)$$

Comparing (1-5) to (1-4) after the transformation (1-3)

$$[G'] = [\Gamma][G][\Gamma]^T \qquad (1\text{-}6a)$$
$$[C'] = [\Gamma][C][\Gamma]^T \qquad (1\text{-}6b)$$
$$[L'^{-1}] = [\Gamma][L^{-1}][\Gamma]^T \qquad (1\text{-}6c)$$

<u>EXAMPLE 1-2</u>

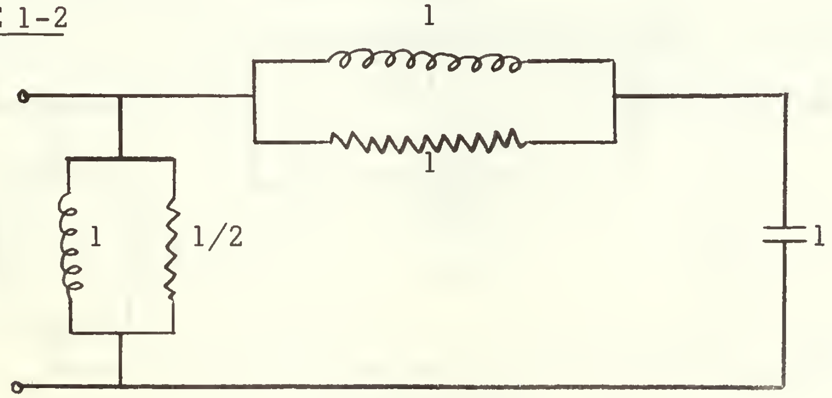

Fig. 1-2a

Consider the R, L, C network of Fig. 1-2a, with the input impedance

$$Z(s) = \frac{S^3 + S^2 + S}{3S^3 + 4S^2 + 3S + 1}$$

$$[G] = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[L^{-1}] = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[C] = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Take $[\Gamma] = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$; where 2 and -2 are chosen arbitrarily.

Then:

$$[G'] = [\Gamma][G][\Gamma]^T = \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix}$$

$$[L'^{-1}] = [\Gamma][D][\Gamma]^T = \begin{bmatrix} 2 & -2 \\ -2 & 4 \end{bmatrix}$$

$$[C'] = [\Gamma][C][\Gamma]^T = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$$

The new network is shown in Fig. 1-2b. The input impedance is unchanged.

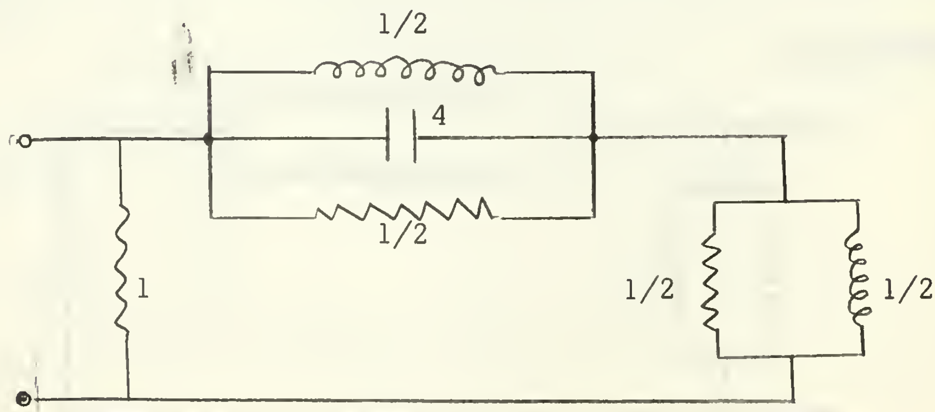

Fig. 1-2b

It can be seen from example 1-2 that the new network has one more element than the original. The circuit configuration and type of elements connected between corresponding nodes is also different.

Guillemin [3] has shown that an equivalent network can be produced by multiplying the same rows and columns of the admittance matrix by the same constant. He has also shown that by using a technique which seems to be similar to linear programming, an equivalent network having all positive elements may be produced from a network with a negative element. The following example will illustrate the above points.

EXAMPLE 1-3

Consider the resistive network of Fig. 1-3a.

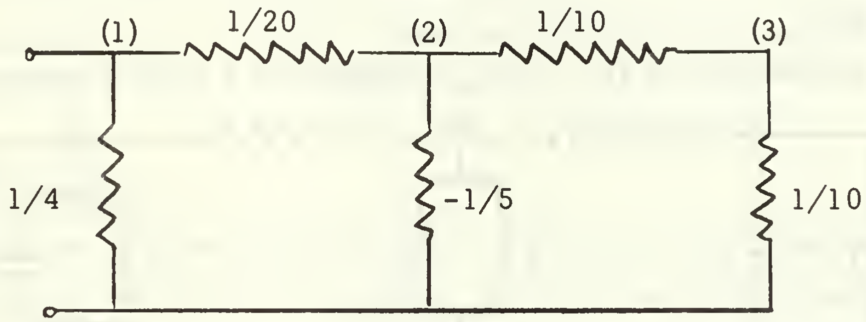

Fig. 1-3a

The network admittance matrix is

$$[G] = \begin{bmatrix} 60 & -20 & 0 \\ -20 & 25 & -10 \\ 0 & -10 & 20 \end{bmatrix}$$

the input impedance $Z_{in} = \frac{1}{40}$ ohms.

If we multiply the second row and second column by 6/5 the new admittance matrix is:

$$[G'] = \begin{bmatrix} 60 & -24 & 0 \\ -24 & 36 & -12 \\ 0 & -12 & 20 \end{bmatrix}$$

The new circuit is shown in Fig. 1-3b.

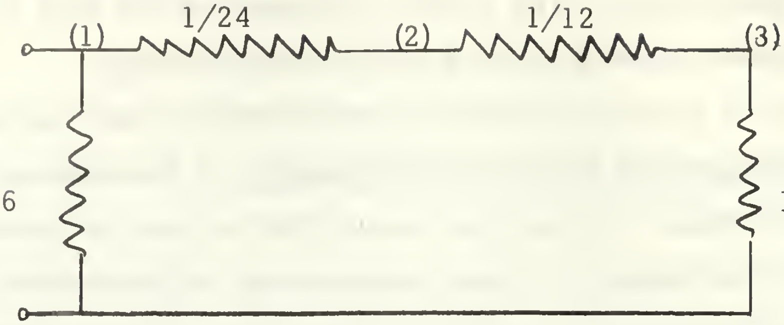

Fig. 1-3b

$$Z_{in} = \frac{1}{40} \text{ ohms}$$

The following example will illustrate the invariance of transfer impedance.

EXAMPLE 1-4

Consider the network of Fig. 1-4a, connected to a current source.

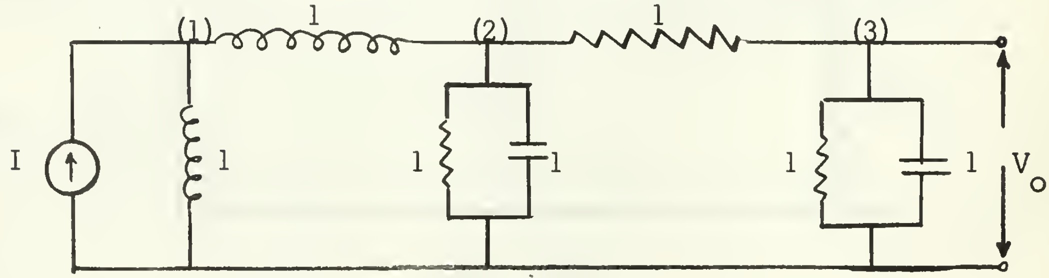

Fig. 1-4a

The resistances are in ohms, capacitances in farads and inductances in henries.

$$[G]=\begin{bmatrix}0 & 0 & 0\\0 & 2 & -1\\0 & -1 & 2\end{bmatrix} \qquad [C]=\begin{bmatrix}0 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \qquad [L^{-1}]=\begin{bmatrix}2 & -1 & 0\\-1 & 1 & 0\\0 & 0 & 0\end{bmatrix}$$

The transformation matrix $[\Gamma]$ is arbitrarily chosen as:

$$[\Gamma]=\begin{bmatrix}1 & 0 & 0\\0 & \frac{3}{4} & 0\\0 & 0 & 1\end{bmatrix}$$

Then:

$$[G'] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{9}{2} & -\frac{3}{2} \\ 0 & -\frac{3}{2} & 2 \end{bmatrix} \qquad [C'] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{9}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [L'^{-1}] = \begin{bmatrix} 2 & -\frac{3}{2} & 0 \\ -\frac{3}{2} & \frac{9}{4} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The circuit diagram of the new network is shown in Fig. 1-4b. The transfer impedance has remained invariant.

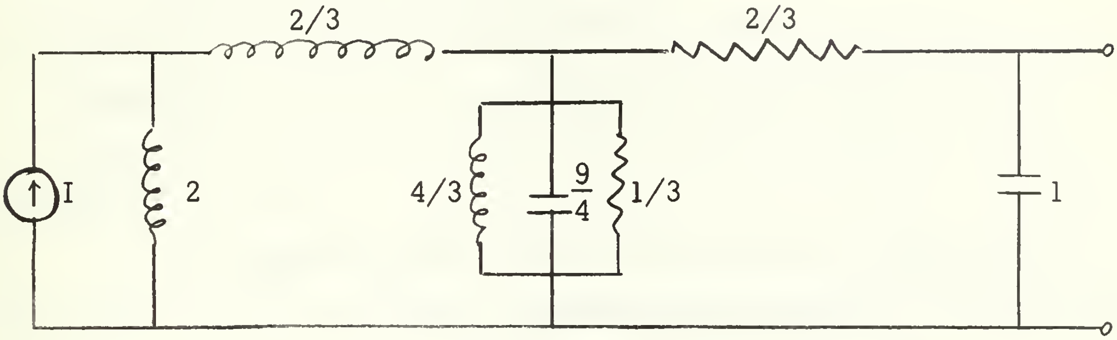

Fig. 1-4b

1.3 Realizability

It has been shown by Guillemin [3] that if the transformation matrix [Γ] is real and non-singular, the positive definite character of the stored energy functions and loss function of the network being transformed remains unchanged. Consequently, under the condition that [Γ] is real and non-singular, a passive network under the transformation of (1-3) is always transformed into another passive network. J.D. Schoeffler [5] has demonstrated that the transformed network can always be realized by imbedding the original network in an array of interconnected ideal transformers. However, the realization of transformed network as a transformerless network with positive elements only presents particular problems and imposes some restrictions on the values of the elements of the transformation matrix [Γ]. These restrictions and allowable ranges for the values of the elements of [Γ] such that the transformed network can be realized with positive R, L, C elements and no mutual inductance is discussed later.

After the transformation is performed and the admittance matrix of the new network is obtained, it is possible to determine whether the new network can be realized by positive R, L, C elements. The following are the necessary and sufficient conditions for [Y'] to be realizable with positive R, L, C elements, Howitt [2].

(1) The off diagonal elements of each of the matrices [G'], [C'] and [L'^{-1}] has to be less than or equal to zero, i.e.:

$$g'_{ij} \leqslant 0 \qquad \text{for all } i \neq j \tag{1-7a}$$

$$C'_{ij} \leqslant 0 \qquad \text{for all } i \neq j \tag{1-7b}$$

$$L'^{-1}_{ij} \leqslant 0 \qquad \text{for all } i \neq j \tag{1-7c}$$

(2) The sum of all the rows of each of the matrices [G], [C] and [L^{-1}] has to be non-negative, i.e.:

$$\sum_{j=1}^{n} g'_{ij} \geqslant 0 \tag{1-8a}$$

$$\sum_{j=1}^{n} C'_{ij} \geqslant 0 \tag{1-8b}$$

$$\sum_{j=1}^{n} L'^{-1}_{ij} \geqslant 0 \tag{1-8c}$$

Conditions (1-7) and (1-8) can be checked by a computer after the transformation is performed.

1.4 Transformation of the Branch Matrices

Since the elements of a network are explicitly presented in the branch admittance or impedance matrices, it is sometimes advantageous to perform the transformation on the branch matrices.

The branch current and voltage matrices of a network can be written as:

$$I_b = \begin{bmatrix} I_{b1} \\ \cdot \\ \cdot \\ I_{bi} \\ \cdot \\ \cdot \\ I_{bn} \end{bmatrix} \qquad\qquad V_b = \begin{bmatrix} V_{b1} \\ \cdot \\ \cdot \\ V_{bi} \\ \cdot \\ \cdot \\ V_{bn} \end{bmatrix}$$

where I_{bi} and V_{bi} represent the current through and the voltage across branch i respectively. The equilibrium conditions of the network can then be written as

$$[V_b] = [Z_b][I_b] \tag{1-6a}$$

or

$$[I_b] = [Y_b][V_b] \tag{1-6b}$$

where $[Z_b]$ is the branch impedance matrix formed as follows:

Each diagonal element Z_{ii} is the total impedance in branch i.

Each off diagonal element Z_{ij} is the mutual inductance common to branches i and j and

$$[Y_b] = [Z_b]^{-1} \tag{1-7}$$

If the loop current matrix of the network is defined by

$$[I] = \begin{bmatrix} I_1 \\ \cdot \\ \cdot \\ I_i \\ \cdot \\ \cdot \\ I_m \end{bmatrix}$$

where I_i is the current in loop i then $[I]$ and $[I_b]$ are related by

$$[I_b] = [\alpha]^T[I] \tag{1-8}$$

where $[\alpha]$ is the network tie set matrix formed as follows.

$$\text{Each element } \alpha_{ij} = \begin{cases} +1 & \text{if } I_{bi} \text{ and } I_i \text{ are in the same direction} \\ -1 & \text{if } I_{bi} \text{ and } I_i \text{ are in the opposite direction} \\ 0 & \text{if branch i is not included in loop i} \end{cases}$$

Similarly if the cut set voltage matrix $[V]$ of a network is defined by

$$[V] = \begin{bmatrix} V_1 \\ \cdot \\ \cdot \\ V_i \\ \cdot \\ \cdot \\ V_k \end{bmatrix}$$

where V_i is a voltage with an arbitrary direction assigned to the branch i of a tree of a network, then

$$[V_b] = [\beta]^T [V] \tag{1-9}$$

where $[\beta]$ is the network cut set matrix formed as follows:

$$\text{Each element } \beta_{ij} = \begin{cases} +1 \text{ if } V_{bi} \text{ and } V_i \text{ are in the same direction} \\ -1 \text{ if } V_{bi} \text{ and } V_i \text{ are in the opposite direction} \\ \;\;0 \text{ if branch i is another branch of the tree} \\ \qquad \text{which includes } V_i \end{cases}$$

Schoeffler [4] has shown that if a matrix $[\Pi]$ is formed such that

$$[\Pi] = [\beta]^T [\Gamma][\alpha] \tag{1-10}$$

where $[\Gamma]$ is the transformation matrix defined in section 1.2, then instead of applying $[\Gamma]$ to the network admittance matrix, we can apply $[\Pi]$ to the network branch admittance matrix $[Y_b]$.

The branch admittance matrix $[Y'_b]$ of the equivalent network is then given by

$$[Y'_b] = [\Pi][Y_b][\Pi]^T \tag{1-11}$$

The advantage of this method is that the elements of the networks are readily seen from $[Y'_b]$ and the need for transformers in the synthesis of the equivalent network is apparent from presence of off diagonal elements in $[Z'_b]$. The disadvantage lies in the fact that $[\Pi]$ has to be formed from equation (1-10) which means that cut set and tie set matrices have to be constructed before $[\Pi]$ can be formed.

1.5 Continuously Equivalent Networks

J.D. Schoeffler [5] has developed a method whereby a set of equivalent networks is produced as a function of an independent variable x. The elements of each member of the set of equivalent networks are functions of the independent variable x. For any value of this variable a network equivalent to the original network is obtained.

Consider that the original $[Y]$ matrix is Y_o. Define a new matrix Y(x) whose elements are some functions of x such that

$$Y(0) = Y_o$$

Choose $\underline{\Gamma}$ such that: (where $[U]$ is the unit matrix)

$$[\Gamma] = [U] + [B]\Delta x \tag{1-12}$$

now

$$\begin{aligned}[Y'(x)] &= [\Gamma][Y(x)][\Gamma]^T \\ &= ([U]+[B]\Delta x)\,[Y(x)]\,([U]+[B]\Delta x)^T \\ &= [Y(x)] + ([Y(x)][B]^T + [B][Y(x)])\,\Delta x \\ &\qquad + [B][Y(x)][B]^T \Delta x^2\end{aligned}$$

If Δx is small then $[Y'(x)]$ will differ from $[Y(x)]$ by a small amount.

Define:

$$[\Delta Y(x)] = [Y'(x)] - [Y(x)] \tag{1-13}$$

then Equation (1-10) becomes:

$$\frac{[\Delta Y(x)]}{\Delta x} = [B][Y(x)] + [Y(x)][B] + [B][Y(x)][B]^T \Delta x^2$$

as $\Delta x \longrightarrow 0$

$$\frac{[dY(x)]}{dx} = [B][Y(x)] + [Y(x)][B]^T \tag{1-14}$$

If the elements of B are chosen arbitrarily, Equation (1-11) can be integrated to yield the elements of $[Y(x)]$ as a function of x. Schoeffler integrates Equation (1-11) numerically and changes the assumed values of the elements of $\underline{B}$ as the integration proceeds. In the numerical integration process the values of the elements of B are chosen so that the new matrix $Y'(x)$ is dominant.* Thus the elements of B are actually a point function of x as the integration process proceeds.

For a network composed of R, L and C we can write

$$[Y(x)] = [G(x)] + S[C(x)] + \frac{1}{S}[L^{-1}(x)] \tag{1-15}$$

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*i.e. the realizability conditions of Sec. 1.3 are satisfied.

substituting Equation (1-12) in (1-11) then expanding and equating terms of equal powers of S on both sides we obtain:

$$\frac{d[G(x)]}{dx} = [B][G(x)] + [G(x)][B]^T \tag{1-16a}$$

$$\frac{d[C(x)]}{dx} = [B][C(x)] + [C(x)][B]^T \tag{1-16b}$$

$$\frac{[L^{-1}(x)]}{dx} = [B][L^{-1}(x)] + [L^{-1}(x)][B]^T \tag{1-16c}$$

which can be shown to reduce to the form

$$\frac{d[G(x)]}{dx} = [M][G(x)] \tag{1-17a}$$

$$\frac{d[C(x)]}{dx} = [M][C(x)] \tag{1-17b}$$

$$\frac{d[D(x)]}{dx} = [M][L^{-1}(x)] \tag{1-17c}$$

or

$$\frac{d[Y(x)]}{dx} = [M][Y(x)] \tag{1-18}$$

where [M] is a matrix whose elements are some combination of the elements of [B]. Matrix [M] can be found by solving the matrix equations:

$$[B][G(x)] + [G(x)][B]^T = [M][G(x)] \tag{1-19}$$

or

$$[B][Y(x)] + [Y(x)][B]^T = [M][Y(x)] \tag{1-20}$$

i.e.

$$[M] = [B] + [Y(x)][B]^T[Y^{-1}(x)] \tag{1-21}$$

Substituting $Y(0) = Y_o$ in Equation (1-21) yields matrix [M] which can then be substituted in Equation (1-18). After integration this yields the admittance matrix [Y(x)] of the equivalent networks as function of the independent variable x.

The Diagonal Variable Transformation and Equivalent Networks with Constraints

2.1 Introduction

In generating equivalent networks using Equation(1-3) or in generating continuously equivalent networks by integrating Equation (1-15,) the designer has no criterion for choosing the elements of the transformation matrix $[\Gamma]$ to achieve a design specification such as changing the value of an element connected between two nodes in a given manner or eliminating a given element from the network. The designer is not even able to choose the elements of $[\Gamma]$ such that the transformed network can be realized by positive R, L, C elements with no mutual inductance. In this chapter a simplified transformation matrix, namely a diagonal matrix, is chosen for the transformation. The acceptable range of elements of $[\Gamma]$ such that the transformed network can be realized by positive R, L, C elements only is found. By choosing the elements of $[\Gamma]$ to be exponential functions of a real variable x, i.e. $e^{\lambda x}$, a group of equivalent networks are generated and it is shown that by choosing the values of λ's certain constraints can be imposed on the network. However as we shall see in Chapter 4, the diagonal form can't always generate equivalent networks.

2.2 The Diagonal Transformation and Realizability Conditions

The transformation matrix is considered to be a diagonal matrix with positive elements, i.e.

$$[\Gamma]=[\Gamma]^T = \begin{bmatrix} \gamma_{11} & 0 & . & . & . & 0 \\ 0 & \gamma_{22} & . & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & . & . & . & \gamma_{nn} \end{bmatrix} \qquad (2\text{-}1)$$

where

$$\gamma_{ii} > 0 \qquad \text{for } i = 1 \ldots . n \tag{2-2}$$

The admittance matrix of the transformed network then becomes:

$$[Y'] = [\Gamma][Y][\Gamma] \tag{2-3}$$

In general

$$[Y] = [G] + S[C] + \frac{1}{S}[L^{-1}] \tag{2-4}$$

Substituting (2-4) in (2-3) we obtain

$$[Y'] = [\Gamma][G][\Gamma] + S[\Gamma][C][\Gamma] + \frac{1}{S}[\Gamma][L^{-1}][\Gamma] \tag{2-5}$$

Since in general [Y'] will also be composed of R, L and C then we can write

$$[Y'] = [G'] + S[C'] + \frac{1}{S}[L'^{-1}] \tag{2-6}$$

Comparing (2-6) and (2-5) we can write:

$$[G'] = [\Gamma][G][\Gamma] \tag{2-7a}$$

$$[C'] = [\Gamma][C][\Gamma] \tag{2-7b}$$

$$[L'^{-1}] = [\Gamma][L^{-1}][\Gamma] \tag{2-7c}$$

A typical term of transformed matrices [G'] , [C'] and $[L'^{-1}]$ is of the form

$$g'_{ij} = \gamma_{ii}\,\gamma_{jj}\,g_{ij} \tag{2-8}$$

By assumption of Equation (2-2) all the γ 's are positive. Hence the sign of the elements of the transformed matrices [G'] , [C'] and $[L'^{-1}]$ remain unchanged. If the original network was made up of passive elements the off diagonal elements of the [G] , [C] and $[L^{-1}]$ matrices were negative. Hence the off diagonal elements of the transformed matrices are also negative and one of the realizability conditions stated in section 1.4 namely Equation (1-6) is always satisfied and the circuit elements connected between any two nodes other than node to datum, remain positive after the transformation.

The sum of the i^{th} row of the transformed matrices [G'] , [C'] and $[L'^{-1}]$ are given by the following equations:

$$S'_i(G) = \sum_{j=1}^{n} g'_{ij} = \sum_{j=1}^{n} \gamma_{ii}\gamma_{jj} g_{ij} \qquad (2\text{-}9a)$$

$$S'_i(C) = \sum_{j=1}^{n} C'_{ij} = \sum_{j=1}^{n} \gamma_{ii}\gamma_{jj} C_{ij} \qquad (2\text{-}9b)$$

$$S'_i(L^{-1}) = \sum_{j=1}^{n} L^{-1}_{ij} = \sum_{j=1}^{n} \gamma_{ii}\gamma_{jj} L^{-1}_{ij} \qquad (2\text{-}9c)$$

It is clear from the method of the construction of the admittance matrix that the above sum of the rows corresponds to the value of the element connected between node i and datum, i.e. g'_{io}, C'_{io} and L'^{-1}_{io}.

If the second of the realizability conditions that is Equation (1-7) is to be satisfied we must have:

$$S'_i(G) = \sum_{j=1}^{n} \gamma_{ii}\gamma_{jj} g_{ij} \geqslant 0 \qquad \text{for all } i \qquad (2\text{-}10a)$$

$$S'_i(C) = \sum_{j=1}^{n} \gamma_{ii}\gamma_{jj} C_{ij} \geqslant 0 \qquad \text{for all } i \qquad (2\text{-}10b)$$

$$S'_i(L^{-1}) = \sum_{j=1}^{n} \gamma_{ii}\gamma_{jj} L^{-1}_{ij} \geqslant 0 \qquad \text{for all } i \qquad (2\text{-}10c)$$

Equations (2-10) represent a set of constraints on the elements of the transformation matrix. Since, from Equation (2-2), γ_i is always positive it can be cancelled from the set of Equations (2-10). Thus in general there are 3n linear equations that have to be satisfied, n equations for each of the [G'], [C'] and [L'^{-1}] matrices comprising [Y']. If Equations (2-10) are satisfied with the equal sign then the corresponding element connected between the corresponding node and datum is zero. For example, if $S'_1(G) = g_{10}$ is zero then the conductance connected between node 1 and datum is eliminated from the circuit. From the above we can conclude that there are no set of γ's which would satisfy all of the 3n equations of (2-10) with the equality sign, since this will result in an equivalent network with no node to datum elements, i.e. with infinite input admittance at any port.

The 3n inequalities of the form (2-10) together with the n inequalities of the form (2-2) are a set of 4n inequalities which must be satisfied by any set of γ's to assure the fact that the transformed network is realizable with passive elements only. Obviously these inequalities are all satisfied if all of the γ's are unity, since the original networks results in this case.

From the discussion on Chapter 1 it is obvious that to keep the input imittance between node k and datum constant γ_k must be equal to unity and to keep the transfer impedance between ports k and ℓ constant we must choose γ_k and γ_ℓ to be unity. There are then n-1 or n-2 remaining γ's to be chosen satisfying Equations (2-2) and (2-10). For example, in the case of transfer function equivalence there are a total of (n-2) + 3n = 4n - 2 inequalities to be satisfied by n-2 unknown γ's. These inequalities are considered in the next section to determine if a solution is possible, and if so to ascertain specific numerical ranges for the γ's over which the inequalities are satisfied. These ranges define the domain of the set of γ's for which the transformation of (2-1) generates networks with passive components for which the transfer impedance between ports i and j remains invariant. Obviously a solution cannot always be expected since the transformed network is confined to the class obtainable by the diagonal transformation matrix (2-1). The general case where a complete n x n transformation matrix is used will be discussed in Chapter 3.

2.3 The Domain of the Solutions

Each of 3n inequalities of Equation (2-10) represents a linear constraint on (n-1) or (n-2) γ's depending on whether the input impedance or transfer impedance is to remain invariant under the transformation. When any of the inequalities (2-10) are satisfied with the equality sign the resulting equation represents a hyperplane in (n-2) dimensional γ - space.* This hyperplane divides the (n-2) dimensional γ - space into two distinct regions. The points on one side of the hyperplane satisfy the corresponding inequality. For example, if the inequality $g'_{io} > 0$ is used the equation of the hyperplane is $g'_{io} = 0$. Points on one side of the plane yield g'_{io} greater than zero and points on the other side yield g'_{io} less than zero. When all of the inequalities (2-10) are considered, the domain of the allowable set of γ's is determined by a hypervolume in γ-space bounded by a set of at most (4n-2) hyperplanes, 3n of which are of the form:

$$\sum_{j=1}^{n} \gamma_{jj}\, g_{ij} = 0 \tag{2-11}$$

and the remaining number in the form

$$\gamma_{ii} = 0 \tag{2-12}$$

where $i \neq k$ and $i \neq \ell$ since $\gamma_{kk} = \gamma_{\ell\ell} = 1$ due to the required invariance of the transfer function. Points on the planes representing Equation (2-12) are not acceptable since these points violate the required non-singularity of the transformation matrix (2-1).

Any point on the hypervolume described above represents a set of γ's which would satisfy all of the inequalities of (2-2) and (2-10).

The realizability volume may be determined in a systematic manner as follows. Solve all the possible combinations of the (n-2) simultaneous equations, taken from the set of (4n-2) equations of the form (2-11) and (2-12) for a set of (n-2) γ's, and check whether these values also satisfy the remaining (4n-2) - (n-2) = 3n inequalities which are not used in choosing the (n-2) simultaneous equations. Any set of γ's which satisfy these conditions is a vertex of the realizability volume. The hypervolume is bounded on the sides by any of the hyper surfaces

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* For convenience of writing, the case of transfer impedance equivalence is considered. The case of input impedance equivalence follows in the same manner.

which passes through (n-2) vertices.

In general a total of

$$\frac{(4n-2)\,!}{[\,(4n-2)-(n-2)\,]\,!\,(n-2)\,!} = \frac{(4n-2)!}{3n!\,(n-2)!} \qquad (2\text{-}13)$$

linear simultaneous equations must be solved and checked in the above manner. In practice many of the solutions are trivial and the process may be programmed on a digital computer. After all vertices of the realizability hyper-volume are determined, the boundary hyper-planes may be determined from the simultaneous equations whose intersections determined the vertices. The point where all of the γ's are equal to one always satisfies the realizability conditions and is always included in the realizability hyper-volume.

If there is no set of γ's which would satisfy the Equations (2-2) and (2-10) we can deduce that the transformation of (2-1) will not produce any equivalent network with passive components, and the realizability hyper-volume in this case is reduced to a single point where all of the γ's are unity. If a realizability volume does exist it is possible to remove at least one node to datum element by choosing a set of γ's represented by a point on one of the boundary hyper-planes of the form (2-11).

<u>EXAMPLE 2.1</u>

Consider the RLC network shown in Fig. 2.1,

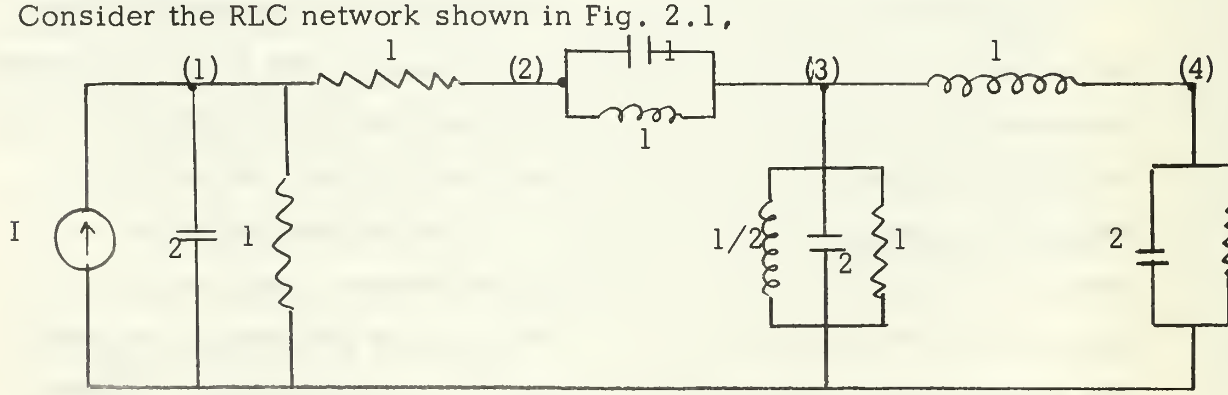

Fig. 2.1

where all the conductances are in mhos, all the capacitances in farads and all the inductances in henries.

$$[G] = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad [C] = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$[L^{-1}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

It is desired to keep the transfer impedance invariant so let:

$$[\Gamma] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \gamma_{22} & 0 & 0 \\ 0 & 0 & \gamma_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transformed matrices $[G']$, $[C']$ and $[L'^{-1}]$ are:

$$[G'] = \begin{bmatrix} 2 & -\gamma_{22} & 0 & 0 \\ -\gamma_{22} & \gamma_{22}^2 & 0 & 0 \\ 0 & 0 & \gamma_{33}^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[C'] = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & \gamma_{22}^2 & -\gamma_{22}\gamma_{33} & 0 \\ 0 & -\gamma_{22}\gamma_{33} & 3\gamma_{33}^2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$[L^{-1}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \gamma_{22}^2 & -\gamma_{22}\gamma_{33} & 0 \\ 0 & -\gamma_{22}\gamma_{33} & 4\gamma_{33}^2 & -\gamma_{33} \\ 0 & 0 & -\gamma_{33} & 1 \end{bmatrix}$$

The inequalities (2-10) in this case are 12 inequalities but only 8 of them contain the variables γ_2 and γ_3 and of these only 6 are distinct and are tabulated below.

From [G'] we have

(a) $2 - \gamma_{22} \geq 0$

(b) $-1 + \gamma_{22} \geq 0$

From [C'] we have

(c) $\gamma_{22} - \gamma_{33} \geq 0$

(d) $-\gamma_{22} + 3\gamma_{33} \geq 0$

From $[L'^{-1}]$ we have

(e) $-\gamma_{22} + 4\gamma_{33} - 1 \geq 0$

(f) $-\gamma_{33} + 1 \geq 0$

In the γ-plane the above inequalities define regions as shown in Fig. 2.2. Taking into account the Equation(2-2)which states

$$\gamma_{22} > 0 \qquad\qquad \gamma_{33} > 0$$

the realizability hypervolume in this two dimensional example will be a four sided polygon shown shaded in Fig. 2.2.

To solve the above problem with a digital computer we must solve all the combinations of the six equations obtained from the inequalities a, b, c, d, e and f two at a time.

Table 2.1 presents all the possible solutions of the group of two simultaneous equations together with the letter representing the solutions on the γ-plane of Fig. 2.2. The solutions which yield valid intersections which satisfy all of the other inequalities are indicated by the asterisks and are the vertices of the realizability area. The boundaries of the volume are given by the lines representing a, b, e and f.

TABLE 2.1

EQUATIONS PAIR	SOLUTIONS γ_{22}	γ_{33}	INTERSECTION POINT
a b	none	none	none
a c	2	2	A
a d	2	2/3	B
a e*	2	3/4	C*
a f*	2	1	D*
b c*	1	1	E*
b d	1	1/3	F
b e*	1	1/2	G*
b f*	1	1	E*
c d	0	0	0
c e	1/3	1/3	H
c f*	1	1	E*
d e	3	1	K
d f	3	1	K
e f	3	1	K

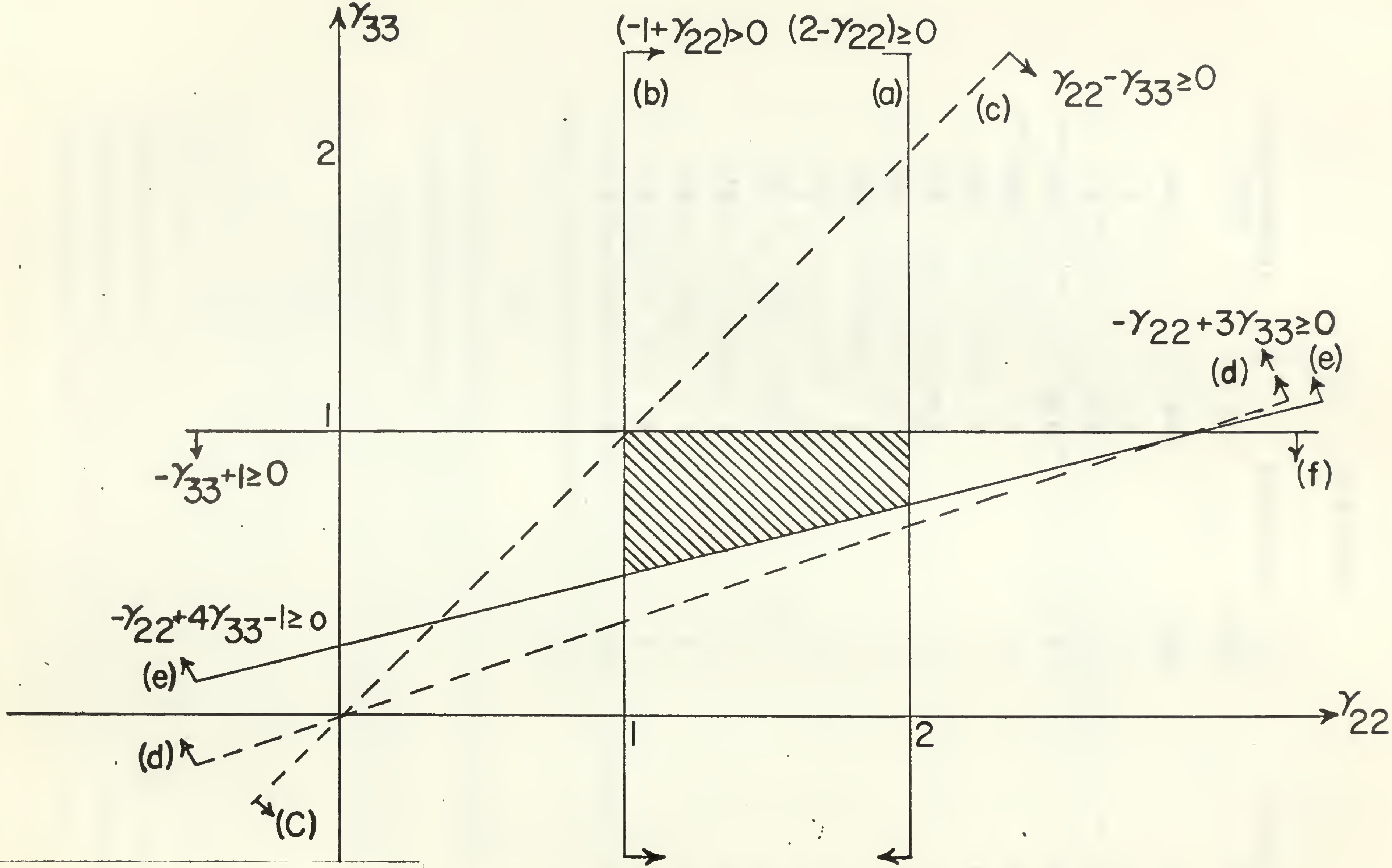

Fig. 2.2 Realizability regions in the γ-plane.

[redacted]

2.4 The Exponential Searching Function

If a volume of realizability does exist then for each point in this volume there exists an equivalent network. These equivalent networks can conveniently be investigated by letting the elements of our transformation matrix $[\Gamma]$ to be exponential functions of an independent real variable x. Thus let

$$\gamma_{ii} = e^{\lambda_{ii} x} \tag{2-14}$$

where λ_{ii} is a constant which may be chosen arbitrary or in accordance with some additional constraint placed upon the desired network.

The exponential function of Equation (2-14) satisfies the requirement of Equation (2-2) in that it is greater than zero for all finite values of the independent real variable x. Hence the transformation matrix $[\Gamma]$ remains real and non-singular for all finite values of x. Moreover when $x = 0$, $\gamma_{ii} = 1$, and for this value of the variable x the original network is reproduced.

When the exponential functions of Equation (2-14) are used as elements of the transformation matrix, $[\Gamma]$ becomes

$$[\Gamma(x)] = \begin{bmatrix} e^{\lambda_{11}x} & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & e^{\lambda_{22}x} & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & e^{\lambda_{nn}x} \end{bmatrix} \tag{2-15}$$

A typical element connected between two nodes in the transformed network becomes:

$$g'_{ij} = g_{ij}\, e^{(\lambda_{ii} + \lambda_{jj})x} \tag{2-16}$$

And a typical node to datum element becomes:

$$g'_{io} = \sum_{j=0}^{n} g_{ij} = \sum_{j=1}^{n} g_{ij}\, e^{(\lambda_{ii} + \lambda_{jj})x} \qquad (2\text{-}17)$$

And a typical equation of the hyperplane used in determining the realizability volume becomes:

$$\sum_{j=1}^{n} g_{ij}\, e^{\lambda_{ii} x} = 0 \qquad (2\text{-}18)$$

g'_{ij} and g'_{io} can easily be plotted by a digital computer as a function of x. The range of x for which all of the 3n equations of the form of Equation (2-17) remain non zero is the range for which transformation of (2-15) yields equivalent networks with positive R, L, C elements.

Differentiating (2-15) with respect to x yields:

$$\frac{d\,[\Gamma(x)]}{dx} = [\Gamma][\lambda] = [\lambda][\Gamma] \qquad (2\text{-}19)$$

where:

$$[\lambda] = \begin{bmatrix} \lambda_{11} & 0 & . & . & . & 0 \\ 0 & \lambda_{22} & . & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & . & . & . & \lambda_{nn} \end{bmatrix} \qquad (2\text{-}20)$$

Differentiating (2-3) with respect to x yields:

$$\frac{d[Y']}{dx} = \frac{d[\Gamma]}{dx}[Y][\Gamma] + [\Gamma][Y]\frac{d[\Gamma]}{dx} \qquad (2\text{-}21)$$

Substituting (2-19) in (2-21) we obtain:

$$\frac{d[Y']}{dx} = [\lambda][Y'] + [Y'][\lambda] \tag{2-22}$$

Thus the use of the exponential search function for generating equivalent networks is the same as integrating Equation (2-22) with the initial condition that $[Y'] = [Y]$ when $x = 0$.

Comparing Equation (2-22) with Equation (1-11) we observe that the two are identical with $[B]$ replaced by $[\lambda]$.

The choice of the λ's may be made to impose additional constraints upon the transformed network as follows.

(a) To keep the transfer impedance between port k and the output voltage between nodes ℓ and m constant (no common node between input and output) set $\lambda_{kk} = \lambda_{\ell\ell} = \lambda_{nn} = 0$.

(b) To keep the admittance between nodes i and j invariant under the transformation set $\lambda_{ii} = -\lambda_{jj}$. From Equation (2-16) it can be seen that in this case $g'_{ij} = g_{ij}$.

(c) From Equation (2-16)

$$g'_{ij} = g_{ij}\, e^{(\lambda_{ii} + \lambda_{jj})\, x} \tag{2-23}$$

and

$$g'_{k\ell} = g_{k\ell}\, e^{(\lambda_{kk} + \lambda_{\ell\ell})\, x} \tag{2-24}$$

Dividing (2-23) to (2-24) we obtain

$$g'_{ij} / g'_{k\ell} = g_{ij} / g_{k} \quad e^{[(\lambda_{ii} + \lambda_{jj}) - (\lambda_{kk} + \lambda_{\ell\ell})]x}$$

Hence to keep the ratio of the admittance connected between nodes i and j to the admittance connected between nodes k and ℓ constant set $(\lambda_{ii} + \lambda_{jj}) = (\lambda_{kk} + \lambda_{\ell\ell})$.

(d) Since the circuit elements between nodes i and j increase or decrease monotonically with x depending upon the sum ($\lambda_{ii} + \lambda_{jj}$), the λ's can be chosen so that a specified component changes more rapidly or slowly compared with others over the range of x.

The use of the exponential search function and constraint (b) for choosing the λ's is demonstrated in example 2.2.

EXAMPLE 2.2

Consider the circuit of example 2.1.

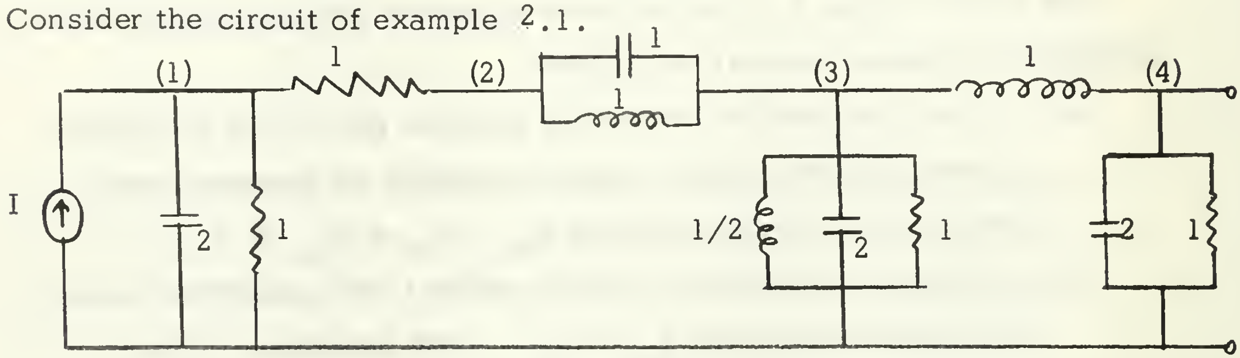

Fig. 2.3

It is desired to keep the transfer impedance as well as the admittance connected between nodes 2 and 3 constant.

From Fig. 2.2 it can be seen that since x = 0 corresponds to point E, to stay in the area of realizability we have to choose λ_{22} and λ_{33} such that γ_{22} increases and γ_{33} decreases with x. Hence λ_{33} must be negative and λ_{22} must be positive, i.e. $\lambda_{22} = -\lambda_{33}$ = constant the value of the constant depends on the desired rate of change of the circuit elements. Since at the moment we are not concerned with rate of change of elements we choose

$$\gamma_{22} = -\gamma_{33} = 1$$

The [G'] , [C'] and [L'^{-1}] matrices would then become:

$$[G'] = \begin{bmatrix} 2 & -e^{x} & 0 & 0 \\ -e^{x} & e^{2x} & 0 & 0 \\ 0 & 0 & e^{-2x} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[C'] = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & e^{2x} & -1 & 0 \\ 0 & -1 & 3e^{-2x} & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$[L'] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & e^{2x} & -1 & 0 \\ 0 & -1 & 4e^{-2x} & -e^{-x} \\ 0 & 0 & -e^{-x} & 1 \end{bmatrix}$$

The new circuit elements become:

$g'_{10} = 2 - e^{x}$	$c'_{10} = 2$
$g'_{12} = e^{x}$	$c'_{20} = e^{2x} - 1$
$g'_{20} = e^{2x} - e^{x}$	$c'_{23} = 1$
$g'_{30} = e^{-2x}$	$c'_{30} = 3e^{-2x} - 1$
$g'_{40} = 1$	$c'_{40} = 2$

$L'_{20} = 1/(e^{2x} - 1)$

$L'_{23} = 1$

$L'_{30} = 1/(4e^{-2x} - e^{-x} - 1)$

$L'_{34} = e^{x}$

$L'_{40} = 1/(1 - e^{-x})$

The variation of the conductances, capacitances and the inverse inductances with the independent variable x are shown in Figs. 2.4, 2.5 and 2.6 respectively.

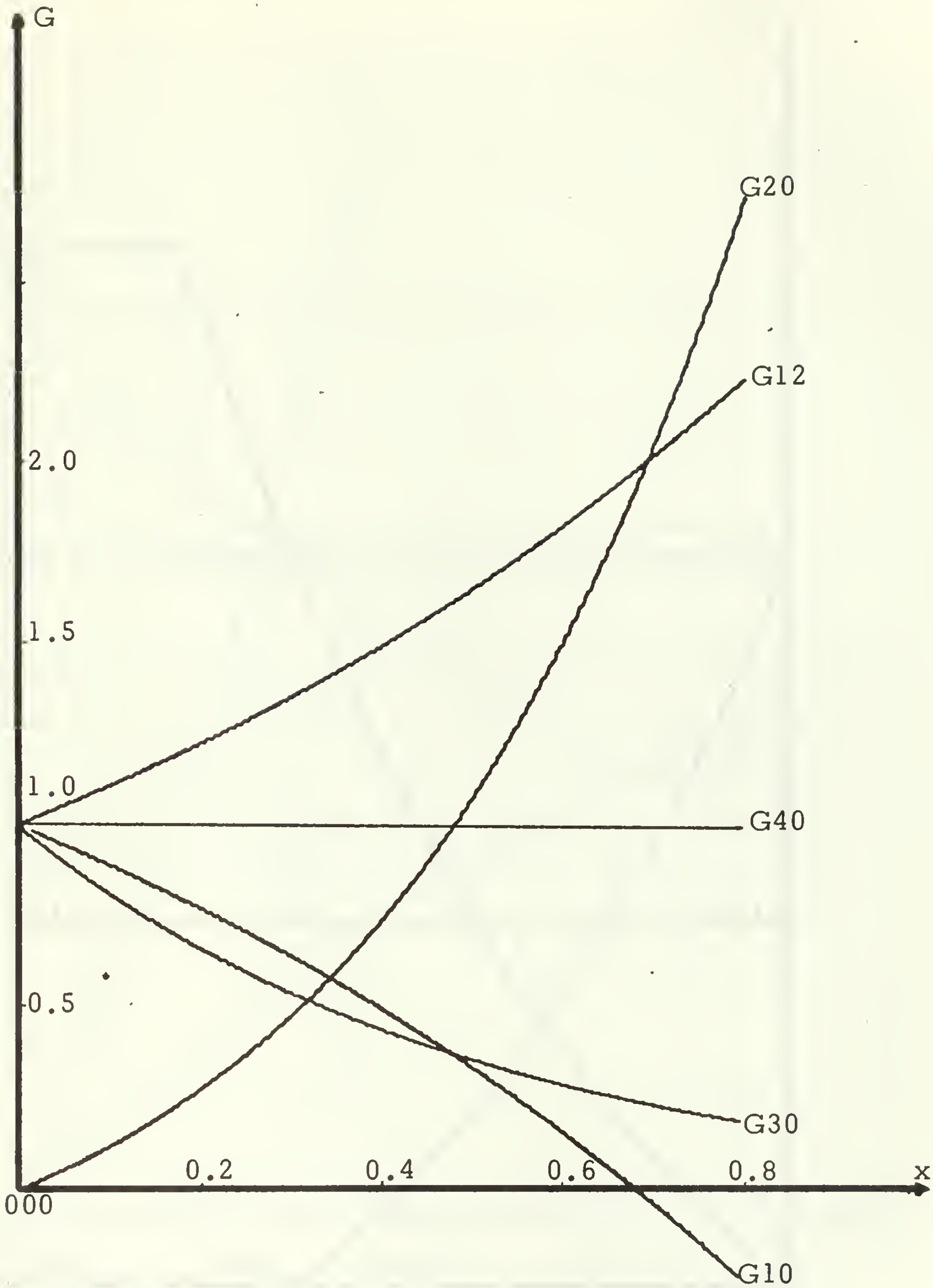

Fig. 2.4 Variation of Conductances with x, Example 2.1

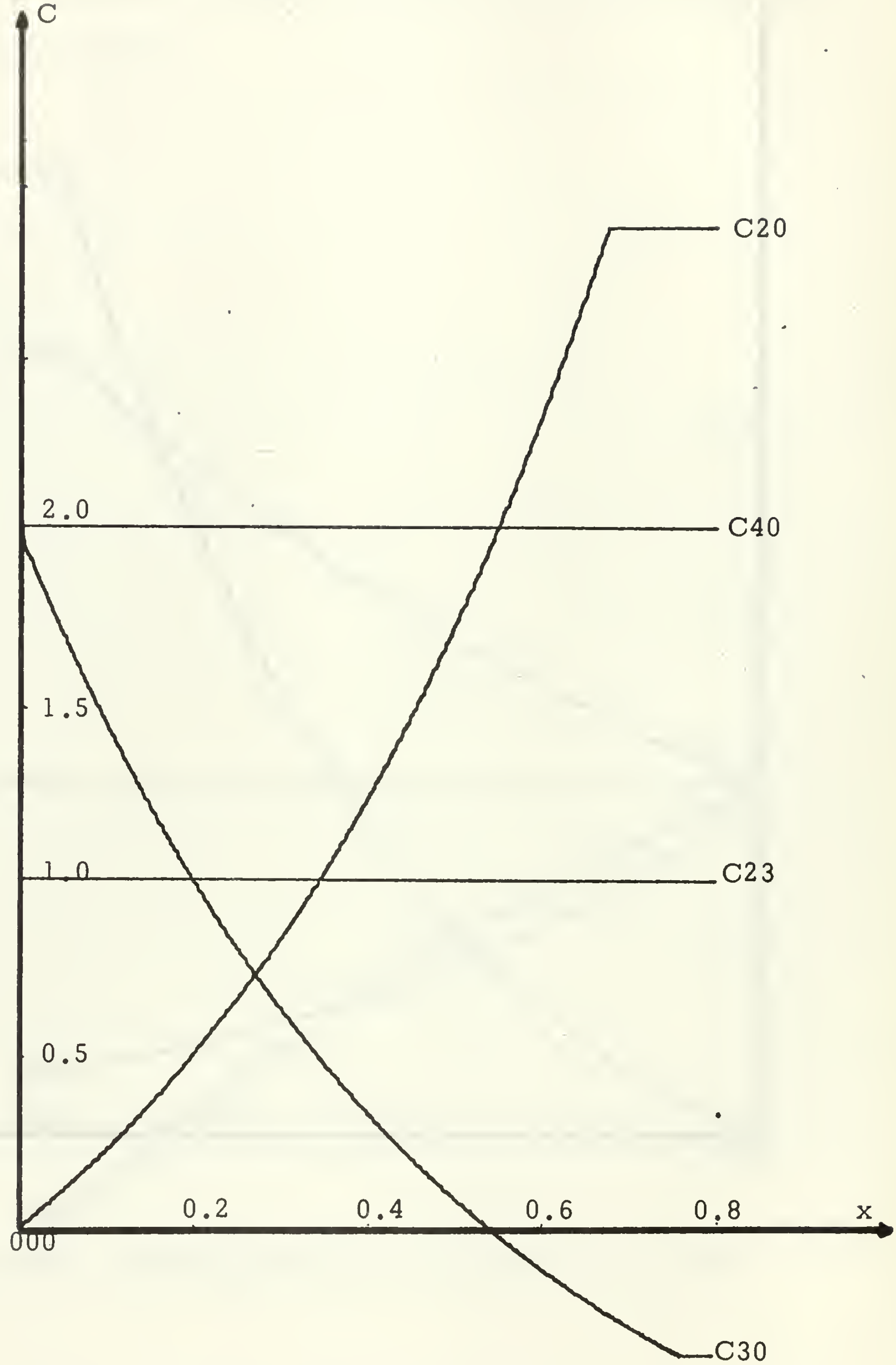

Fig. 2.5 Variation of Capacitances with x, Example 2.1

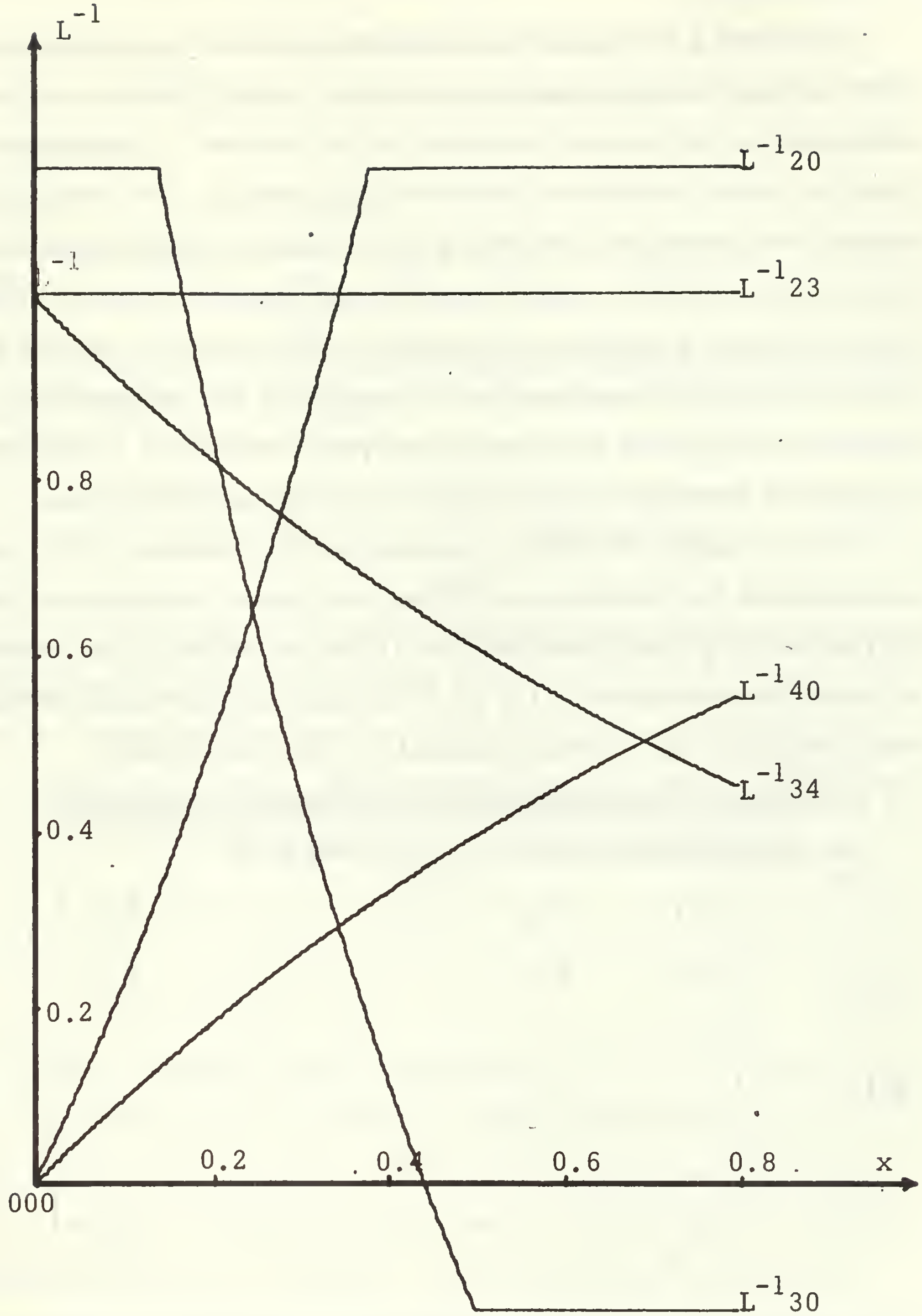

Fig. 2.6 Variation of Inverse Inductances with x, Example 2.1

The General Transformation

3.1 Introduction

In Chapter 2 a diagonal transformation matrix was used and it was shown that the realizability hyper-volume, when it exists, can be determined by the manner described in that chapter. The diagonal transformation matrix however is limited in application. For example it is obvious from Equation (2-8) that if no conductance existed between nodes i and j in the original network, none of the equivalent networks generated will have a conductance between nodes i and j. But the simplicity of the transformation (2-3), linearity of the realizability equations (2-10), and the ease of applying constraints on the transformed networks warranted the use of the diagonal transformation.

In this chapter the full n x n transformation matrix $[\Gamma]$ is used. The equations for the elements of the transformed networks are obtained. The realizability conditions and the allowable range for the elements of the transformation matrix $[\Gamma]$ so as to yield to equivalent networks with positive R, L, C circuit elements is then discussed.

3.2 The General Transformation and Realizability Conditions

The transformation matrix Γ is chosen to be

$$[\Gamma] = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdot & \cdot & \cdot & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \cdot & \cdot & \cdot & \gamma_{2n} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \gamma_{n1} & \cdot & \cdot & \cdot & \cdot & \gamma_{nn} \end{bmatrix} \tag{3-1}$$

A typical element of the conductance, capacitance, and inverse inductance matrices $[G']$, $[C']$ and $[L'^{-1}]$ of the equivalent networks produced will then be of the form:

$$g'_{ij} = \sum_{\ell,k=1}^{n} \gamma_{i\ell}\gamma_{jk}\, g_{\ell k} \tag{3-2}$$

Since the original network is made up of bilateral elements, the [G], [C] and [L^{-1}] matrices are symmetric. Hence Equation (3-2) can be written as:

$$g'_{ij} = \sum_{k=1}^{n} g_{kk}\gamma_{ik}\gamma_{jk} + \sum_{\substack{p=1\\ q=p+1}}^{n} g_{pq}\left[\gamma_{ip}\gamma_{jq} + \gamma_{iq}\gamma_{jp}\right] \tag{3-3}$$

Where i = j, Equations (3-3) yield the diagonal elements of the matrices [G'], [C'] and [L'^{-1}]. Thus:

$$g'_{ii} = \sum_{k=1}^{n} g_{kk}\gamma_{ik}^{2} + 2\sum_{\substack{p=1\\ q=p+1}}^{n} g_{pq}\gamma_{ip}\gamma_{iq} \tag{3-4}$$

We recall from Chapter 1 that realizability with passive components required that the off diagonal elements of the [G'], [C'] and [L'^{-1}] be zero or negative and that the sum of terms in any row of the above matrices be zero or positive. Thus realizability with passive elements in the case of the full transformation matrix [Γ] requires that

$$g'_{ij} \leq 0 \qquad \text{for } i \neq j \tag{3-5}$$

$$g'_{io} = \sum_{j=1}^{n} g'_{ij} \geq 0 \tag{3-6}$$

For an n port network each of the matrices [G'], [C'] and [L'^{-1}] is a symmetrical n x n matrix. Hence we have in general 3n(n-1)/2 inequalities of the form (3-5) and 3n inequalities of the form (3-6) to be satisfied. Since the transformation matrix [Γ] need not be symmetrical it contains n^2 elements. The invariance of the input or transfer impedance requires that one or two columns of the transformation matrix $[\Gamma]$ be unit vectors (i.e. zeros everywhere except on the diagonal position). Hence if the invariance of the transfer impedance is required a total of $3n(n+1)/2$ inequalities have to be satisfied by $(n^2-2n) = n(n-2)$ γ 's if the transformed network is to be realizable with passive components.

Examination of (3-3), (3-4), (3-5) and (3-6) shows that the inequalities (3-5) and (3-6) which have to be satisfied are non-linear functions of the γ 's. Hence the method used in Chapter 2 for the diagonal transformation matrix is not applicable here, since the equations to be solved are not simultaneous linear equations. To be able to apply the method of Chapter 2 to the transformation of (3-1) used in this chapter, we will linearize the inequalities of (3-5) and (3-6) as shown in the next section. It must be noted that in the case of transformation with the full transformation matrix of (3-1), in contrast to the case of diagonal transformation matrix, the γ 's can take both positive and negative values.

3.3 Linearization

We know from the discussion in Chapter 2 that the inequalities (3-5) and (3-6) are satisfied when $\gamma_{ii} = 1$ and $\gamma_{ij} = 0$ for $i \neq j$. This will result in the transformation matrix $[\Gamma]$ to be the identity matrix $[I]$ and the transformation will yield the original network.

We can linearize the inequalities (3-5) and (3-6) by expanding the matrix $[Y']$ in a Taylor series expansion about the point $\gamma_{ii} = 1$ and $\gamma_{ij} = 0 \quad i \neq j$ as shown below:

From (1-3) we have

$$[Y'] = [\Gamma][Y][\Gamma]^T \tag{3-7}$$

or from (1-6a) we have

$$[G'] = [\Gamma][G][\Gamma]^T \tag{3-8}$$

Hence

$$[\Delta G'] = [\Delta\Gamma][G][\Gamma]^T + [\Gamma][G][\Delta\Gamma]^T$$

$$\therefore [\Delta G'] = \left(\sum_{i,j=1}^{n} \frac{\partial[\Gamma]}{\partial\gamma_{ij}} \Delta\gamma_{ij} \right) [G][I] + [\Gamma][G]$$

$$\times \left(\sum_{l,j=1}^{n} \frac{\partial[\Gamma]}{\partial\gamma_{ji}} \Delta\gamma_{ji} \right) \tag{3-9}$$

But

$$\frac{\partial[\Gamma]}{\partial\gamma_{ij}} \Delta\gamma_{ij} = \begin{bmatrix} 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \Delta\gamma_{ij} & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix}$$

Hence

$$\sum_{\substack{i=1 \\ j=1}}^{n} \frac{\partial\Gamma}{\partial\gamma_{ij}} \Delta\gamma_{ij} = \begin{bmatrix} \Delta\gamma_{11} & \Delta\gamma_{12} & \cdot & \cdot & \cdot & \Delta\gamma_{1n} \\ \Delta\gamma_{21} & \Delta\gamma_{22} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \Delta\gamma_{n1} & \cdot & \cdot & \cdot & \cdot & \Delta\gamma_{nn} \end{bmatrix} = [\Delta\gamma] \tag{3-10}$$

Since we are expanding $[G']$ with a Taylor series about the point $\gamma_{ii} = 1$, $\gamma_{ij} = 0$ $\quad i \neq j$ $\quad$ we can write

$$\gamma_{ii} = 1 + \delta_{ii} \tag{3-11}$$

$$\gamma_{ij} = 0 + \delta_{ij} \tag{3-12}$$

where δ_{ii} and δ_{ij} are incremental change on γ_{ii} and γ_{ij}, i.e. $\delta_{ii} = \Delta\gamma_{ii}$ and $\delta_{ij} = \Delta\gamma_{ij}$ and the transformation of (3-1) with $\delta_{ii} = \delta_{ij} = 0$ yields the original network. The origin of the δ-space corresponds to $[\Gamma]$ being the identity matrix.

Substituting (3-11) and (3-12) in 3-9 we can write

$$[\delta] = [\Delta\gamma] = \sum_{i,j=1}^{n} \frac{\partial[\Gamma]}{\partial\gamma_{ij}} \Delta\gamma_{ij} \tag{3-13}$$

Substituting (3-13) in (3-9) we obtain

$$[\Delta G'] = [\delta][G][\Gamma]^T + [\Gamma][G][\delta]^T \tag{3-14}$$

Expanding $[G']$ with the Taylor series around $[G]$ yields

$$[G'] \simeq [G']|_o + [\Delta G']|_o \tag{3-15}$$

where $[G']|_o$ and $[\Delta G']|_o$ denotes the matrices of $[G']$ and $[\Delta G']$ where $\gamma_{ii} = 1$ and $\gamma_{ij} = 0 \quad i \neq j$ or where $\delta_{ii} = \delta_{ij} = 0$.

From (3-8)

$$[G']|_o = [G] \tag{3-16}$$

Since $[\Gamma]|_o = [\Gamma]^T|_o = [U]$ where $[U]$ denotes the identity matrix then from (3-14) we have

$$[\Delta G']|_o = [\delta][G] + [G][\delta]^T \tag{3-17}$$

Substituting (3-17) and (3-16) in (3-15) yields

$$[G'] = [G] + [\delta][G] + [G][\delta]^T \tag{3-18}$$

Hence a typical term of the $[G']$, $[C']$ and $[L'^{-1}]$ matrices would be

$$g'_{ij} = g_{ij} + \sum_{k=1}^{n} (g_{kj}\delta_{ik} + g_{ik}\delta_{jk}) \tag{3-19}$$

where only the first two terms of the Taylor series are taken.

Equation (3-19) will enable us to linearize the inequalities of (3-5) and (3-6) and hence apply the method of Chapter 2 to obtain the area of realizability and the range of γ's for which the transformation of (3-1) yields equivalent networks with passive elements. In essence this procedure identifies a small region in δ- space, any point of which corresponds to a set of γ's for which the transformation of (3-1) may be applied to yield equivalent networks with passive elements. Once this region is defined a searching function such as the exponential searching function of Chapter 2 can be used to generate equivalent networks whose circuit elements are functions of an independent variable such as x and the complete inequalities of (3-5) and (3-6) investigated to yield equivalent networks with passive elements.

3.4 The Exponential Searching Function

We use an exponential searching function for the elements of the transformation matrix $[\Gamma]$ in the following form:

$$\gamma_{ii} = e^{\lambda_{ii} x} \qquad (3\text{-}20)$$

$$\gamma_{ij} = 1 - e^{\lambda_{ij} x} \qquad i \neq j \qquad (3\text{-}21)$$

Approximating (3-20) and (3-21) by the first two terms of their Taylor series expansion and comparing with Equations (3-11) and (3-12) respectively we obtain

$$\lambda_{ii} x = \delta_{ii} \qquad (3\text{-}22)$$

$$-\lambda_{ij} x = \delta_{ij} \qquad (3\text{-}23)$$

Equations (3-22) and (3-23) in conjunction with the linearized form of Equations (3-5) and (3-6) can be used to choose values of the λ's and the range of x, as best illustrated by the following example.

EXAMPLE 3.1

Consider the RC network of Fig. 3.1.

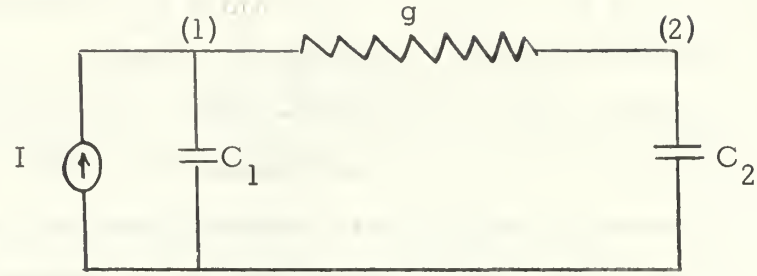

Fig. 3.1

It is desired to find the equivalent networks containing passive elements which have the same input impedance as the above network.

The [G] and [C] matrices of the above network are:

$$[C] = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \qquad [G] = g\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Starting with the diagonal transformation matrix

$$[\Gamma] = \begin{bmatrix} 1 & 0 \\ 0 & \gamma_{22} \end{bmatrix}$$

The [G'] and [C'] matrices will be

$$[G'] = g\begin{bmatrix} 1 & -\gamma_{22} \\ -\gamma_{22} & \gamma_{22}^2 \end{bmatrix} \qquad [C'] = \begin{bmatrix} C_1 & 0 \\ 0 & C_2\gamma_{22}^2 \end{bmatrix}$$

The realizability conditions are

$$1 - \gamma_{22} \geqslant 0$$

$$-1 + \gamma_{22} \geqslant 0$$

$$C_2\,\gamma_{22}^2 \geqslant 0$$

It is obvious that the only value of γ_{22} which satisfies the above equations is $\gamma_{22} = 1$ which means that the transformation matrix is the identity matrix. Hence the diagonal form of the transformation matrix will not generate any equivalent network.

Now we use the full transformation matrix

$$[\Gamma] = \begin{bmatrix} 1 & \gamma_{12} \\ 0 & \gamma_{22} \end{bmatrix}$$

The [G'] and [C'] matrices will be

$$[G'] = g \begin{bmatrix} 1-2\gamma_{12} + \gamma_{12}^2 & \gamma_{22}\gamma_{12} - \gamma_{22} \\ \gamma_{22}\gamma_{12} - \gamma_{22} & \gamma_{22}^2 \end{bmatrix}$$

$$[C'] = \begin{bmatrix} C_1 + C_2\gamma_{12}^2 & C_2\gamma_{12}\gamma_{22} \\ C_2\gamma_{12}\gamma_{22} & C_2\gamma_{22}^2 \end{bmatrix}$$

The realizability conditions (3-5) and (3-6) become:

(a) $g'_{10} = (1-2\gamma_{12} + \gamma_{12}^2 + \gamma_{12}\gamma_{22} - \gamma_{22})\, g \geq 0$

(b) $g'_{20} = (\gamma_{22}\gamma_{12} - \gamma_{22} + \gamma_{22}^2)\, g \geq 0$

(c) $g'_{12} = (\gamma_{22}\gamma_{12} - \gamma_{22})\, g \leqslant 0$

(d) $C'_{10} = C_1 + C_2(\gamma_{12}\gamma_{22} + \gamma_{12}^2) \geqslant 0$

(e) $C'_{20} = C_2(\gamma_{12}\gamma_{22} + \gamma_{22}^2) \geqslant 0$

(f) $C'_{12} = C_2\gamma_{12}\gamma_{22} \leqslant 0$

The linearized form (3-19) of the above equations are

(a) $-\delta_{12} - \delta_{22} \geqslant 0$

(b) $\delta_{12} + \delta_{22} \geqslant 0$

(c) $1 + \delta_{22} - \delta_{12} \geqslant 0$

(d) $C_1 + C_2\,\delta_{12} \geqslant 0$

(e) $C_2\,(\delta_{12} + 2\,\delta_{22} + 1) \geqslant 0$

(f) $C_2\,\delta_{12} \leqslant 0$

Each of the above inequalities when satisfied with equal signs will be presented by a line on δ_{12}, δ_{22} space as shown in Fig. 3.2. The linearized realizability region reduces to the portion of the line $\delta_{22} + \delta_{12} = 0$ in the second quadrant of the δ space shown on Fig. 3.2.

Using the exponential search function, Equations (3-20) and (3-21) become

$$\gamma_{22} = e^{\lambda_{22}x}.$$

$$\gamma_{12} = 1 - e^{\lambda_{12}x}$$

Equations (3-22) and (3-23) become

$$\lambda_{22} x = \delta_{22}$$

$$-\lambda_{12} x = \delta_{12}$$

From the conditions of realizability for this case we have $\delta_{22} = -\delta_{12} =$ constant. The value of the constant determines the rate of change of the numerical values of the circuit element. Since we are not concerned at this moment about the rate of change of elements we arbitrarily choose $\lambda_{22} = \lambda_{12} = 1$ for $x > 0$.
Hence

$$\gamma_{22} = e^{x}$$

$$\gamma_{12} = 1 - e^{x}$$

Substituting the above values in the expressions for g's and c's yields

$$g'_{10} = 0 \qquad C'_{10} = C_1 + C_2(1 - e^{x})$$

$$g'_{20} = 0 \qquad C'_{20} = C_2 e^{x}$$

$$g'_{12} = -ge^{2x} \qquad C'_{12} = C_2 e^{x}(1 - e^{x})$$

From the above equations it can be seen that the range of x for which the realizability conditions are satisfied is given by the two equations

$$C_1 + C_2(1 - e^{x}) \geq 0$$

$$C_2 e^{x}(1 - e^{x}) \leq 0$$

Hence

$$1 \leq e^{x} \leq 1 + \frac{C_1}{C_2}$$

or

$$0 \leq x \leq \ln \frac{C_1 + C_2}{C_2}$$

The equivalent network is shown in Fig. 3.3 below and the variation of the network elements with x are shown in Fig. 3.4 for the case of $g = 1$, $C_1 = C_2 = 1$.

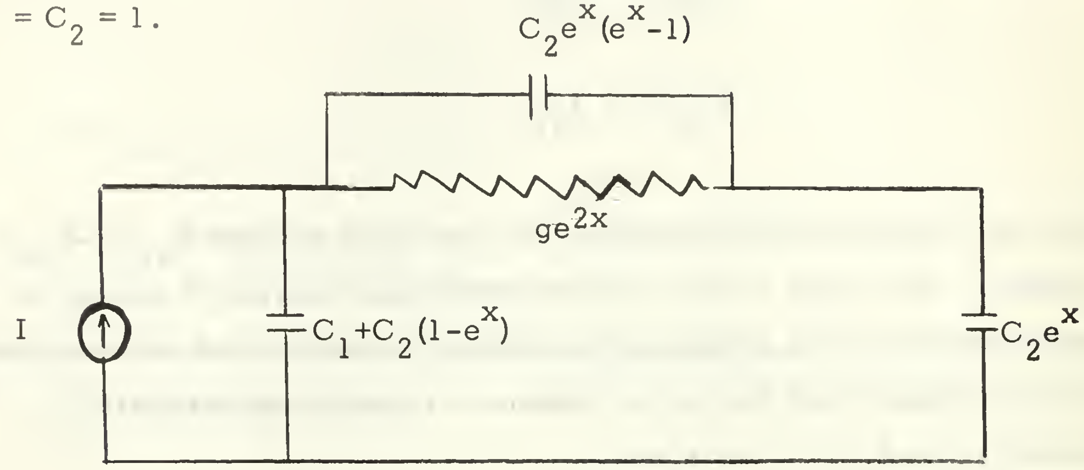

Fig. 3.3

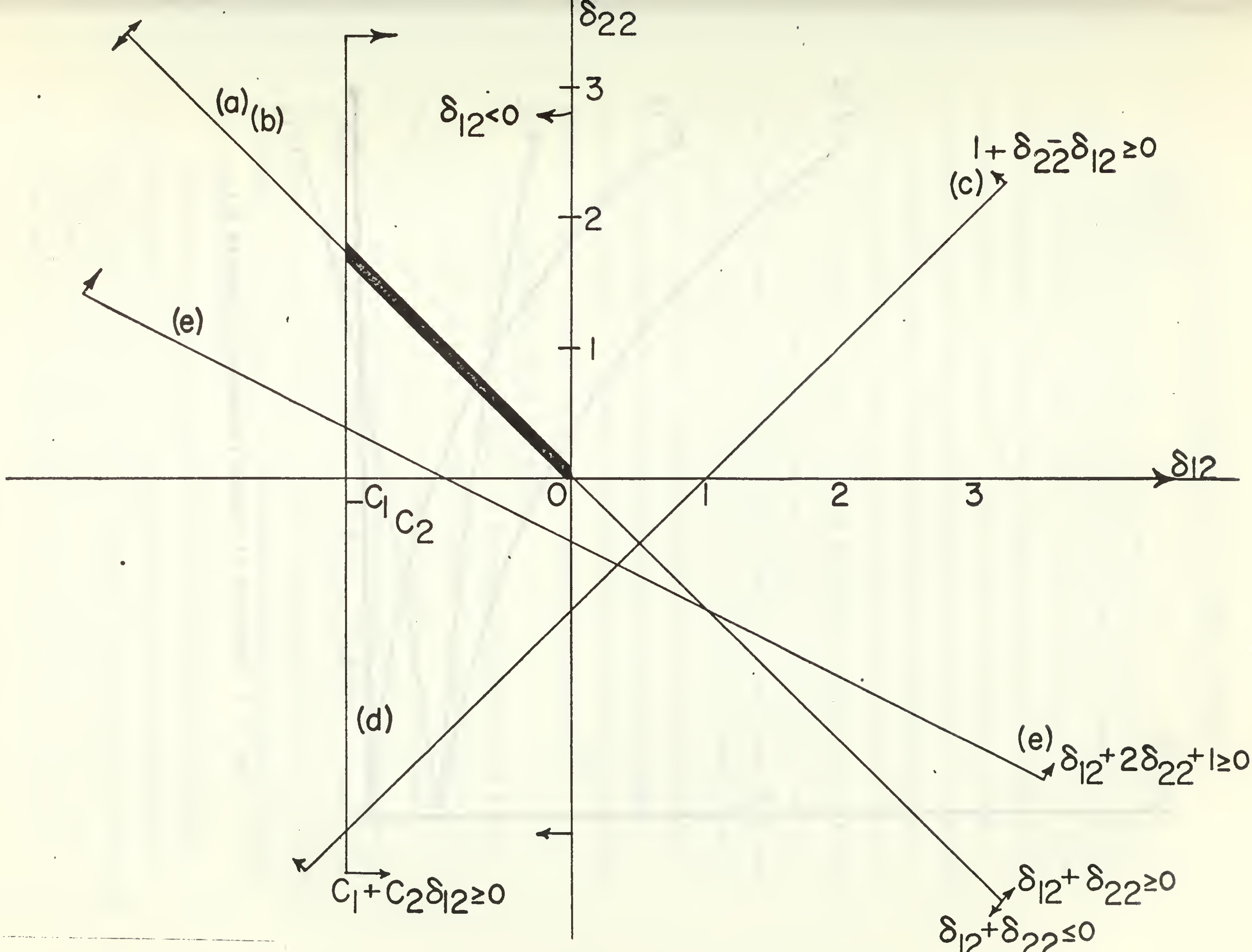

Fig. 3.2 Linearized realizability conditions in the δ plane.

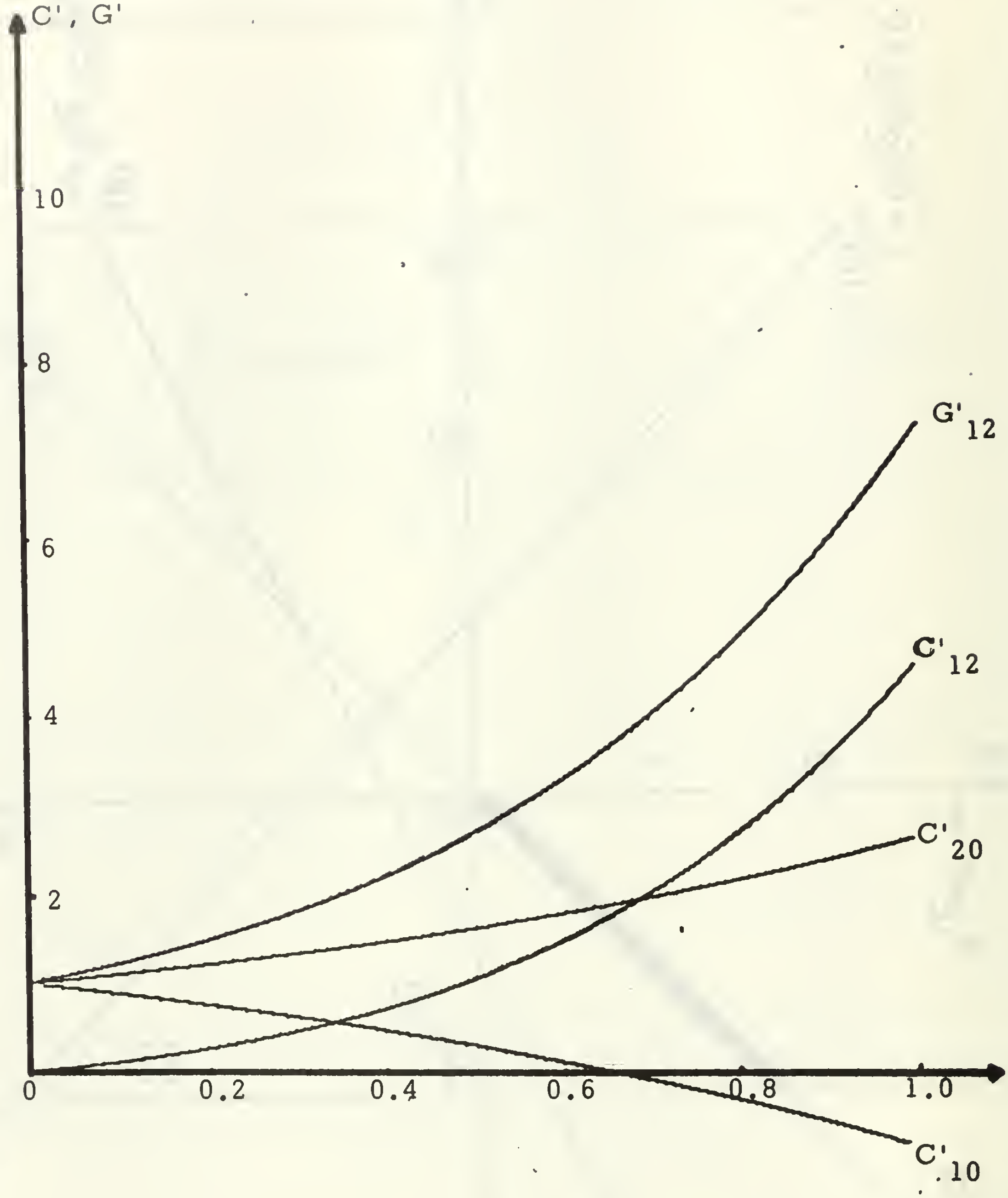

Fig. 3.4 Variation of Circuit Elements of Example 3.1

Conclusion and Suggestions for Further Research

4.1 Conclusion

In this paper the congruent transformation is applied to the admittance matrix of an n+1 node common datum network to generate equivalent networks which have the same input or transfer admittance. It is shown that a transformation matrix whose elements are exponential functions of a real variable x, such as $e^{\lambda x}$, can be used to generate a group of equivalent networks whose circuit elements are functions of the variable x. The range of variation of the elements of the transformation matrix required to generate equivalent networks with passive circuit elements is found by solving a number of sets of equations called the realizability equations. If a diagonal transformation matrix is used these equations are linear but when a full transformation matrix is used these equations are non-linear, and have to be linearized. If a diagonal transformation matrix, whose elements are exponential functions of the real variable x is used, then certain constraints can be placed upon the generated equivalent networks by choosing certain relations between the constant factors (λ's) in the exponent of the exponential function $e^{\lambda x}$.

4.2 Suggestions for Further Research

It is believed that the subject of equivalent networks has a great potential for application in the field of network synthesis and analysis. The following are some ideas for further research on this subject.

(a) It may be possible to find the general form of a transformation which would take a given form of a synthesised network to another form, For example it may be possible to find a transformation matrix $[\Gamma]$ which transforms the Foster form of a synthesised network to Cauer's form for the same input impedance.

(b) The effect of the presence of mutual inductance in a network on the transformation can be investigated with the aim of transforming a network with mutual inductance to a network without mutual inductance.

(c) Finally, from curves of Fig. 2.4, 2.5 and 2.6, it can be seen that if the values of the components of the network vary according to the curves shown, then the transfer impedance and input impedance remain invariant, i.e. the multiparameter sensitivity of the input or transfer impedance is then zero. Hence, multiparameter sensitivity can be studied from the point of view of equivalent networks.

Bibliography

1. Cauer, W., "Vierpole", Elek. Nachr. Tech., Vol. 6
2. Howitt, N., "Group Theory and the Electric Circuit", Phys. Rev., Vol. 37, pp. 1583-1595, 1931
3. Guillemin, E.A., Synthesis of Passive Networks, John Wiley & Sons, Inc., pp. 141-176, 1957
4. Schoeffler, J.D., "Foundation of Equivalent Network Theory", Technical Report #2, Engineering Division, Case Institute of Technology, Dec. 1961
5. Schoeffler, J.D., "The Synthesis of Minimum Sensitivity Networks", IEEE Transactions on Circuit Theory, Vol. CT-11, pp. 271-276, June 1964
6. Reza, F., "Some Topological Considerations in Network Theory", IRE Transaction on Circuit Theory, pp. 30-42, March 1958
7. Browne, E.T., Determinents and Matrices, 1st Edition
8. VanValkenburg, M.E., Introduction to Modern Network Synthesis, John Wiley & Sons, Inc., 1962
9. Calahan, D.A., Network Synthesis, Vol. 2, Hayden, pp. 82-94, 1964

Appendix I

Invariance of Input Impedance

It was stated in Chapter 1 that if the column i of the transformation matrix $[\iota]$ is a unit vector (zero everywhere except for a one at the i^{th} row) then the input impedance at port i, that is Z_i, remains invariant under the transformation. In this appendix a proof of this fact based on matrix portioning is presented.

Consider an n+1 node network with a current source connected between each of the n nodes and the datum node as shown in Fig. I-1.

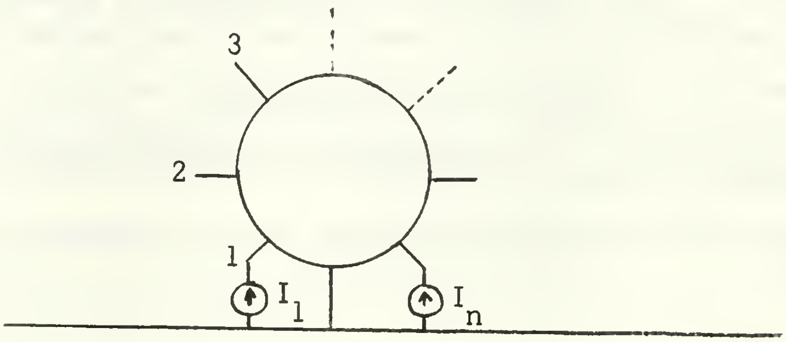

Fig. I-1

The equilibrium equation of the network can be written in matrix form as

$$[I] = [Y][V] \tag{I-1}$$

with $[I]$, $[Y]$ and $[V]$ as defined in Chapter 1.

Expanding the matrix Equation (I-1) and using Crammer rule, the voltage V_i between node i and datum is given by

$$V_i = \frac{\Delta_{i1}}{\Delta} I_1 + \frac{\Delta_{i2}}{\Delta} I_2 + \dots + \frac{\Delta_{ii}}{\Delta} I_i + \dots \frac{\Delta_{in}}{\Delta} I_n \tag{I-2}$$

where:

Δ = the determinent of the admittance matrix $[Y]$

Δ_{ij} = the minor formed from Δ by deleting row i and column j

If the network is excited only at node i by a current source I_i then:

$I_j = 0 \quad$ for $i = 1 \dots n \quad i \neq j$

Hence $$V_i = \frac{\Delta_{ii}}{\Delta} I_i \tag{I-3}$$

and

$$Z_i = \frac{\Delta_{ii}}{\Delta} \tag{I-4}$$

The invariance of the input impedance at port i under the congruent transformation

$$[Y'] = [\Gamma][Y][\Gamma]^T \tag{I-5}$$

with column i of $[\Gamma]$ being a unit vector would be proved if we can show that the ratio Δ_{ii} / Δ of Equation (I-4) remains invariant under transformation (I-5).

For simplicity of proof we consider the invariance of the input impedance at node 1. Since by renumbering the nodes we can call any node node 1, there is no loss of generality and the proof would be acceptable for the invariance of the input impedance at any port.

Consider a real non-singular transformation matrix $[\Gamma]$ where column 1 is a unit vector

$$[\Gamma] = \begin{bmatrix} 1 & \gamma_{12} & \cdot & \cdot & \cdot & \gamma_{1n} \\ 0 & \gamma_{22} & \cdot & \cdot & \cdot & \gamma_{2n} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \gamma_{n2} & \cdot & \cdot & \cdot & \gamma_{nn} \end{bmatrix} \tag{I-6}$$

The admittance matrix $[Y']$ of the transformed network is given by

$$[Y'] = [\Gamma][Y][\Gamma]^T \tag{I-7}$$

where $[Y]$ and $[Y']$ are as defined in Chapter 1.

Since the determinant of the product of matrices is equal to the product of the determinants $[\Gamma]$ then:

$$\Delta' = |\Gamma|^2 \Delta \tag{I-8}$$

where Δ' = determinant of $[Y']$

and $|\Gamma|$ = determinant of $[I]$

We can partition the $[\Gamma]$, $[\Gamma]^T$, $[Y]$, $[Y']$ matrices as shown below.*

$$\left[\begin{array}{c|cccc} y'_{11} & \cdot & \cdot & \cdot & y'_{1n} \\ \hline \cdot & & & & \\ \cdot & & X' & & \\ \cdot & & & & \\ y'_{n1} & & & & \end{array}\right] = \left[\begin{array}{c|cccc} 1 & \gamma_{12} & \cdot & \cdot & \cdot\ \gamma_{1n} \\ \hline 0 & & & & \\ \cdot & & \alpha & & \\ \cdot & & & & \\ 0 & & & & \end{array}\right] \times$$

$$\left[\begin{array}{c|cccc} y_{11} & \cdot & \cdot & \cdot & y_{1n} \\ \hline \cdot & & & & \\ \cdot & & & & \\ \cdot & & X & & \\ y_{n} & & & & \end{array}\right] \times \left[\begin{array}{c|cccc} 1 & 0 & \cdot & \cdot & \cdot\ 0 \\ \hline a_{12} & & & & \\ \cdot & & & & \\ \cdot & & \alpha^T & & \\ a_{1n} & & & & \end{array}\right] \tag{I-9}$$

Performing the matrix operation we obtain

$$[X'] = [\alpha][X][\alpha]^T \tag{I-10}$$

Hence the admittance matrix of the new network is given by:

---

* y_{ij} $i \neq j$ = the admittance connected between nodes i and j

y_{ii} = the sum of all admittances connected to node i

$$
[Y'] = \left[\begin{array}{c|ccc}
y'_{11} & \cdot \quad \cdot \quad \cdot & & y'_{1n} \\
\hline
\cdot & & & \\
\cdot & & [\alpha][X][\alpha]^T & \\
\cdot & & & \\
y'_{n1} & & &
\end{array}\right] \tag{I-11}
$$

The input impedance at node i of the new network can be written from Equation (I-4) to be

$$
Z'_1 = \frac{\Delta'_{11}}{\Delta'} = \frac{\det.\,([\alpha][X][\alpha]^T)}{\Delta'} \tag{I-12}
$$

Since determinant of product of matrices is equal to the product of the determinants [7] we can write

$$
\det\,([\alpha][X][\alpha]^T) = |\alpha|^2 |X| = |\alpha|^2 \Delta_{11} \tag{I-13}
$$

where $|\alpha|$ = determinant of $[\alpha]$.

Since the 1st column of $[I]$ is a unit vector

$$
|\Gamma| = |\alpha| \tag{I-14}
$$

Substituting (I-14) in (I-13) we obtain

$$
\det\,([\alpha][\alpha][\alpha]^T) = |\Gamma|^2 \Delta_{11} \tag{I-15}
$$

Substituting (I-8) and (I-15) in (I-12) yields

$$
Z'_1 = \frac{|\Gamma|^2 \Delta_{11}}{|\Gamma|^2 \Delta} = \frac{\Delta_{11}}{\Delta}
$$

Hence

$$
Z'_1 = Z_1 \tag{I-16}
$$

Appendix II

Computer Program for Example 2.1

```
..JOB0184F,ARDALAN
      PROGRAM EQTRANF1
      DIMENSIONX(500),G10(500),G12(500),G20(500),G30(500),G40(500),C10(
     1500),C20(500),C23(500),C30(500),C40(500),P20(500),P23(500),P30(50
     2),P34(500),P40(500),ITITLE(12),GA2(500),GA3(500)
      GM2=1.
      GM3=-1.
      X(1)=0.
      A=0.8
      DO 10 I=1,500
      G12(I)=EXPF(GM2*X(I))
      G10(I)=2.-G12(I)
      G20(I)=EXPF(2.*GM2*X(I))-G12(I)
      G30(I)=EXPF(2.*GM3*X(I))
      G40(I)=1.
      C10(I)=2.
      C23(I)=EXPF((GM2+GM3)*X(I))
      C20(I)=EXPF(2.*GM2*X(I))-C23(I)
      C30(I)=3.*EXPF(2.*GM3*X(I))-C23(I)
      C40(I)=2.
      P23(I)=EXPF((GM2+GM3)*X(I))
      P20(I)=EXPF(2.*GM2*X(I))-P23(I)
      P34(I)=EXPF(GM3*X(I))
      P30(I)=4.*EXPF(2.*GM3*X(I))-P23(I)-P34(I)
      P40(I)=1.-P34(I)
      GA2(I)=EXPF(GM2*X(I))
      GA3(I)=EXPF(GM3*X(I))
      X(I+1)=X(I)+(A/500.)
   10 CONTINUE
      DO 22 I=1,12
   22 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HG10
      CALL DRAW(500,X,G10,1,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 23 I=1,12
   23 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HG12
      CALL DRAW(500,X,G12,2,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 24 I=1,12
   24 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HG20
      CALL DRAW(500,X,G20,2,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 25 I=1,12
   25 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
```

```
      LABLE=4HG30
      CALL DRAW(500,X,G30,2,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 26 I=1,12
   26 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HG40
      CALL DRAW(500,X,G40,3,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 27 I=1,12
   27 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HC10
      CALL DRAW(500,X,C10,1,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 28 I=1,12
   28 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HC23
      CALL DRAW(500,X,C23,2,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 29 I=1,12
   29 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HC20
      CALL DRAW(500,X,C20,2,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 30 I=1,12
   30 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HC30
      CALL DRAW(500,X,C30,2,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 31 I=1,12
   31 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HC40
      CALL DRAW(500,X,C40,3,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 32 I=1,12
   32 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HL23
      CALL DRAW(500,X,P23,1,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 33 I=1,12
   33 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HL20
      CALL DRAW(500,X,P20,2,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 34 I=1,12
   34 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HL34
      CALL DRAW(500,X,P34,2,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 35 I=1,12
   35 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
```

```
      LABLE=4HL30
      CALL DRAW(500,X,P30,2,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 36 I=1,12
   36 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HL40
      CALL DRAW(500,X,P40,3,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      END
      END
```

Appendix III

Computer Program for Example 3.1

```
..JOB0184F,ARDALAN
      PROGRAM NONLINER
      DIMENSION X(500),G10(500),G12(500),G20(500),C10(500),C12(500),C20(
     1500),ITITLE(12)
      A=1.
      GM12=1.
      GM22=1.
      G=1.
      C1=1.
      C2=1.
      X(1)=0.
      DO 10 I=1,500
      G12(I)=G*EXPF((GM22+GM12)*X(I))
      C12(I)=C2*(EXPF(GM22*X(I))*(EXPF(GM12*X(I))-1.))
      C10(I)=C1+C2*(EXPF(GM22*X(I))*(1.-EXPF(GM12*X(I)))+(1.-EXPF(GM12*X
     1(I)))**2)
      C20(I)=-C12(I)+C2*EXPF(2.*GM22*X(I))
      PRINT 15,X(I),G12(I),C10(I),C12(I),C20(I)
   15 FORMAT(1X,5F10.6)
      X(I+1)=X(I)+(A/500.)
   10 CONTINUE
      DO 23 I=1,12
   23 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HG12
      CALL DRAW(500,X,G12,1,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 24 I=1,12
   24 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HC10
      CALL DRAW(500,X,C10,2,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 25 I=1,12
   25 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HC12
      CALL DRAW(500,X,C12,2,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      DO 26 I=1,12
   26 ITITLE(I)=12
      ITITLE(1)=8HARDALAN
      LABLE=4HC20
      CALL DRAW(500,X,C20,3,0,LABLE,ITITLE,0,0,0,0,0,0,5,5,0,LAST)
      END
      END
```

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY

13. ABSTRACT

In this thesis the generation of equivalent networks by means of a congruent transformation using a variable transformation matrix applied to the admittance matrix of an n+1 node, common datum, network is studied. The ranges of the values of the elements of the transformation matrix which yield passive RLC components for the equivalent network without mutual inductances are considered. When the transformation matrix is diagonal the solution involves a set of linear algebraic inequalities. In the general case, when all the elements of the transformation matrix are present, the solution involves non-linear algebraic inequalities. The region of passive component realizability consists of a hyper-volume in the space of the elements of the transformation matrix, and the use of exponential searching functions to generate equivalent networks with design constraints is investigated.

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